Virchow (R.), For. Mem. R.S. Crania Ethnica Americana. Folio. *Berlin* 1892. The Author.

Bronze copy of the Gray Prize Medal of the Marischal College, Aberdeen. P. J. Anderson, Esq.

"On the Method of Examination of Photographic Lenses at the Kew Observatory." By Leonard Darwin, Major, late Royal Engineers. Communicated by Captain Abney, C.B., F.R.S. Received April 13,—Read June 2, 1892.

The Kew Committee of the Royal Society decided, about a year ago, to undertake the examination of photographic lenses, thus adding one more to the already numerous list of tests and certificates issued by the Kew Observatory. Captain Abney was the member of the Committee who originated the idea, and he was requested to organise the system in detail. This he undertook to do, but unfortunately it proved that official duties and his work in connexion with colour vision, &c., made it impossible for him to give the necessary time to the enquiry. In consequence of this, I was asked by the Kew Committee, with the full acquiescence of Captain Abney, to give my assistance in the matter, which I gladly consented to do; eventually the arrangements devolved almost entirely upon myself, acting in co-operation with Mr. Whipple, the Superintendent of the Observatory, and aided by consultations with Captain Abney; but I should add that as to the arguments and discussion in this paper I alone am responsible. A very considerable amount of time and energy was expended by Mr. Whipple and myself before the final recommendations could be made, but now, since the whole of the proposals have received the provisional approval of the Kew Committee, it is open to anyone to get a photographic lens examined at Kew on payment of a small fee.

It is important first to state clearly the general idea which the Kew Committee had in view when they undertook this new work, for, if the methods adopted are judged from any other standpoint, they will, no doubt, be found open to criticism. The object of the Committee was to organise a system by which any one could obtain, on payment, an impartial and authoritative statement of the quality of a lens to be used for ordinary photographic purposes, and that the fee, which had to cover the cost of the examination, should be moderate. This latter consideration acted as a serious restriction, and it was consequently necessary that all the tests should give results of undoubted practical value to the practical photographer; the certificate of examination must be recorded in the way most

generally useful, and in language which could not fail to be understood. A complete scientific investigation of a lens from every point of view would occupy so long a time as to make the necessary fee quite prohibitive, and, moreover, the results would contain much information which would be quite useless to the ordinary user of the lens.

There are undoubted advantages in testing a lens by the examination of negatives made by it, but it may be here stated, once for all, that the question of expense rendered it impossible, for the present, to adopt any photographic method; eye observations alone have to be relied on.

The form of entry is made to state for what special purposes the lens is intended, whether for portrait work, for landscape views, or for copying plans, &c. Every lens for photographic purposes is more or less of a compromise. Great rapidity, great perfection in definition, and power of covering very wide angles are incompatible qualities, and one or other of them must be sacrificed. It is therefore evidently unfair to expect different types of lenses to give equally good results under the same test; for if we select a lens excelling greatly in one of these qualifications, we must deliberately abandon the expectation of its attaining the highest standards in the others. For example, in a portrait lens great rapidity is required, but, on the other hand, a less high standard of definition near the edges of the plate can be tolerated than with a landscape lens. No opinion could possibly be expressed at Kew as to the wisdom of demanding extra perfection or powers in any respect, and it is therefore necessary that the lenses should be, to a certain extent, classified by the parties sending them in for examination.

The smaller the aperture of a lens, the larger will be the field of sharp definition covered by it, and a complete study of a lens would tell us the size of the plate which is properly covered when each of the different stops is used. Considering the restrictions necessarily imposed on the work, such a lengthy examination could not possibly be thought of. Hence, when discussing the programme of tests to be applied at Kew, it was soon evident that the time devoted to the examination of each lens had to be limited by making the person entering it state either the number of the largest stop by which it should be judged, or the size of the plate for which it would be used; on the first supposition the Kew certificate would have given the size of the plate which the lens covered satisfactorily with the named stop; and, on the second supposition, it would have indicated the size of the largest stop that could be used to give results up to a certain standard, or the rapidity of the lens in normal cases when used for the plate of the named size. The latter of these two alternatives has been adopted, because it is considered that the owner or intending purchaser of the lens will, in most cases, have already

decided on the size of the plate he intends to use, and that what he wants to know is whether it is suitable for that plate or not. When further information is desired, the lens may be entered for examination for two or more sizes of plates.

The following is an example of the Certificate of Examination, the part in italics representing the result of the testing of the lens. Reference is made to the pages where a detailed discussion on each test can be found :—

KEW OBSERVATORY, RICHMOND, SURREY.

Certificate of Examination of a Photographic Lens.

1. Number on lens, *3876*. Registered number, *95*.
2. Description, *landscape lens*. Diameter, *1·5* inches.
3. Maker's name, *A. B.*
4. Size of plate for which the lens is to be examined, *6·5* inches by *8·5* inches.
5. Number of reflecting surfaces, *4* (see p. 409).
6. Centering in mount, *good* (see p. 409).
7. Visible defects—such as striæ, veins, feathers, &c., *nil* (see p. 409).
8. Flare spot, *nil* (see p. 409).
9. Effective aperture of stops (see p. 412)—

Number engraved on stop.	Effective aperture. Inches.	*f*/number.	C.I. No.*
No. *7·5*	*1·32*	*f/8·6*	*1/1·38*
No. *10*	*1·19*	*f/9·5*	*1/1·12*
No. *15*	*0·97*	*f/11·7*	*1·35*
No. *25*	*0·75*	*f/15·1*	*2·26*
No. *50*	*0·49*	*f/23*	*5·3*
No.........	....	....	....
No.........	....	....	....

10. Angle of cone of illumination with largest stop = *68°*, giving a circular image on the plate of † *13·2* inches diameter.

    Angle of cone outside which the aperture begins to be eclipsed, with stop C.I. No. *1/1·38*, = *20°*, giving a circular image on the plate of *4·0* inches diameter.

    Diagonal of the plate = *10·7* inches, requiring a field of *51°*.

    Stop C.I. No. *5·3* is the largest stop of which the whole opening can be seen from the whole of the plate (see p. 414).
11. Principal focal length,† = *11·28* inches. Back focus, or length from the principal focus to the nearest point on the surface of the lenses, = *10·4* inches (see p. 418).
12. Curvature of the field, or of the principal focal surface. After focussing † the plate at its centre, movement necessary to bring it into focus for an image *1·5* inches from its centre = *0·02* inch.

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* C.I.—International Congress System. [See p. 413; an explanatory note will accompany certificate.]

† The lens is focussed on a very distant object.

Ditto for an object *3* inches from its centre = *0·04* inch.
" *4·5* " = *0·10* "
" *5* " = *0·15* " (see p. 425).

13. Definition at the centre with the largest stop, *excellent*. C.I. stop No. *1·35* gives *good* definition over the whole of a *6·5* inch by *8·5* inch plate (see p. 429).
14. Distortion. Deflection or sag in the image of a straight line which, if there were no distortion, would run from corner to corner along the longest side of a *6·5* inch by *8·5* inch plate = + 0·*01* inch* (see p. 436).
15. Achromatism. After focussing† in the centre of the field in white light, the movement necessary to bring the plate into focus in blue light (dominant wave-length, 4420), = + 0·*04* inch.‡ Ditto in red light (dominant wave-length, 6250) = −0·*01* inch† (see p. 440).
16. Astigmatism.§ Approximate diameter of disc of diffusion† in the image of a point, with C.I. stop No. —— at —— inches from the centre of the plate = 0·—— inch (see p. 443).
17. Illumination of the field. The figures indicate the relative intensity at different parts of the plate.†

With C.I. stop No. *1/1·38*.			With stop No. *5·3*.	
At the centre............	100	:	Ditto	100
At *3* inches from the centre	*67*	:	Ditto	*82*
At *5·35* "	*28*	:	Ditto	*66*

General Remarks.—*An excellent medium angle rapid objective, practically free from distortion.*

Date of issue ________________

W. HUGO, Observer.
G. M. WHIPPLE, Superintendent.

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* The sag or sagitta here given is considered positive if the curve is convex towards the centre of the plate.

† The lens is focussed on a very distant object.

‡ Positive if movement towards the lens, negative if away from it.

§ The lens is supposed to be perfect in other respects.

Note.—The following is the scale of terms used : excellent, good, fair, indifferent, bad.

In considering and in recording the results of examinations, it has been found convenient to give more exact meanings to certain expressions than have as yet been assigned to them. The following definitions have therefore been adopted at Kew :—

A narrow angle lens means one covering effectively not more than 35°.

A medium angle lens means one covering between 35° and 55°.

A wide angle lens means one covering between 55° and 75°.

An extra wide angle lens means one covering more than 75°.

With regard to the wording of the "General Remarks" in the certificate, it should be remembered that the lens is judged entirely with reference to a plate of named size; the lens is therefore classed as above by the angle of field which is given as the last item but one in test No. 10. If the same lens is examined for plates of different sizes, the certificate would be worded differently in each case.

The C.I. No. of a stop means the number which indicates the intensity of illumination produced by it on the plate according to the system proposed at the International Photographic Congress of 1889 (see p. 413).

The *largest normal stop* means the largest stop that can be used with the lens so as to produce definition up to a selected standard of excellence all over a plate of given size, the objects whose images are seen being all equally distant.

A slow lens means one of which the largest normal stop has a less diameter than has C.I. No. 6.

A moderately rapid lens is one of which the largest normal stop is C.I. No. 6, or larger than that size and less than C.I. No. 2.

A rapid lens is one of which the largest normal stop is C.I. No. 2, or larger than that size and less than C.I. No. 2/3.

An extra rapid lens is one of which the largest normal stop is C.I. No. 2/3, or larger than that size.

For convenience of reference, these definitions will in future accompany the certificate, probably in the form of additional notes.

No doubt most lenses are supplied with stops larger than the ones here called the largest normal stops, even if it is not intended to use smaller plates than those under consideration; this is, of course, very right, for in many cases the photographer will be willing to sacrifice the definition near the edge of the plate for the sake of increased rapidity.

It now remains to be shown in what way the above certificate of examination would be useful to the practical photographer, who has sent his lens to Kew for the purpose of being tested. It may, we think, be assumed that he wants answers to the three following questions:—1st. Is the lens a good one? 2ndly. Does it properly cover the plate of the named size? And 3rdly. What exposure must be given when using the different stops?

With regard to the two first questions, the result of the examination is recorded in such a way that he may either rely on the "General Remarks," or he may form an independent judgment from the results of the tests.

In order to decide himself, from the records in the certificate, whether the lens is generally speaking a good one, he should first look to test No. 13 to see if the definition in the centre of the plate with the largest stop, is "excellent," as should always be the case; he should then consider test No. 15, by which he will see what are the faults introduced by the lens not being properly corrected for chromatic aberration. With regard to the second question, that is to say, when considering whether the plate he intends to use is properly covered or not, he should chiefly look to the results recorded under test No. 13, where is given the size of stop or the rapidity of

the lens for a given standard of definition up to the edge of the plate; if the definition at the centre is "excellent," then any want of definition at the margin will be chiefly due to curvature of the focal surface, or to astigmatism, and therefore the results of tests Nos. 12 and 16 should be considered at the same time as test No. 13. He must also look carefully to the result of test No. 14, which shows the maximum distortion produced in the image; it will depend for what class of work the lens is to be used whether he should consider the amount of curvature in the image of a straight line near the edge of the plate, which will be there indicated, is objectionable or not.

The "General Remarks" are recorded as the result of exactly similar considerations to those discussed above, the experience gained by the examination of lenses of undoubted quality giving an idea of what standard of excellence should be required.

With regard to the third question, as to the exposure to be given with the different stops, it may be hoped before long that the C.I. numbering will be generally adopted by all practical photographers, in which case the results of test No. 9 will give the information required.

In many works on photography the view is expressed that the practical photographer also wishes to know from what point on his lens he should measure or adjust the distance of any object so that by reference to tables, he can obtain definite enlargements or reductions; this is, in fact, urging that the position of the principal planes should be marked on the mounting of all lenses. According to our experience this is a want in reality very seldom felt in practice. The tables are, no doubt, sometimes used to get approximate results, the fine adjustment of scale being afterwards done by measurements on the ground glass; but if the slot between the two lenses of a doublet is used as the point from which the measurements of distance are made, the results will be quite near enough to the truth to serve as a first adjustment, and for this purpose nothing will be gained by marking the exact position of the principal planes; it should, however, be stated that the omission to mark them is merely made in consequence of the necessity felt of minimising in every possible direction the time spent in the examination.

Each test to which the lens is subjected will now be described in detail, together with such discussion as to the reason for its adoption as may appear necessary.

The first four headings of the certificate deal with the numbering of the lens, the maker's name, the size of plate for which the lens is to be examined, &c.; and, as these do not form part of the results of the examination, no remarks are necessary with regard to them.

5. *Number of Reflecting Surfaces.*

In most cases the number of reflecting surfaces of glass is known at once from the type of lens, but, if in doubt, a simple experiment will settle the point; the room is darkened, and the reflection of a lamp is observed in the lenses; each of the surfaces of the lenses will give one direct reflected image, and the number can thus easily be counted. The amount of light which reaches the photographic plate decreases with an increased number of lenses, because of this reflection, and this circumstance should not be forgotten in estimating the suitability of a lens for any special purpose. Surfaces merely separated by Canada balsam reflect little light, and need not be considered from this point of view.

6. *Centering in Mount.*

Two different errors might be described under this heading: either (1) the optical axis of a perfect lens may not coincide with the axis of the mounting, or (2) the axes of the different lenses of a doublet or triplet may not all be in the same straight line. As to the first of these errors, we believe it would never be sufficient to have any appreciable effect on the practical value of a lens, and therefore no test for it is considered necessary. With regard to the second error, Wollaston's test is the only one applied; this consists of looking at the flame of a lamp or candle *through* a compound lens, and noting if all the different images of the light as seen by successive reflections from the surfaces of the glass can be brought into line by a suitable movement of the whole lens, which should be the case if the component lenses are arranged about a common axis.

It may be remarked that the nodal points may be shifted away from the mechanical axis of the lens in consequence of either of the abovementioned errors, and also, on the other hand, that the second error may exist—that the axes of the component lenses of a doublet may not be coincident—and yet one or both of the nodal points may conceivably be found on the mechanical axis of the mounting; it follows, therefore, that to estimate the distance between the nodal points and the mechanical axis, which has been suggested as a means of detecting any want of centering, does not answer that purpose very well.

7. *Visible Defects, such as Striæ, Veins, Feathers, &c.*

Under this heading any faults detected by a careful inspection are given.

8. *Flare Spot.*

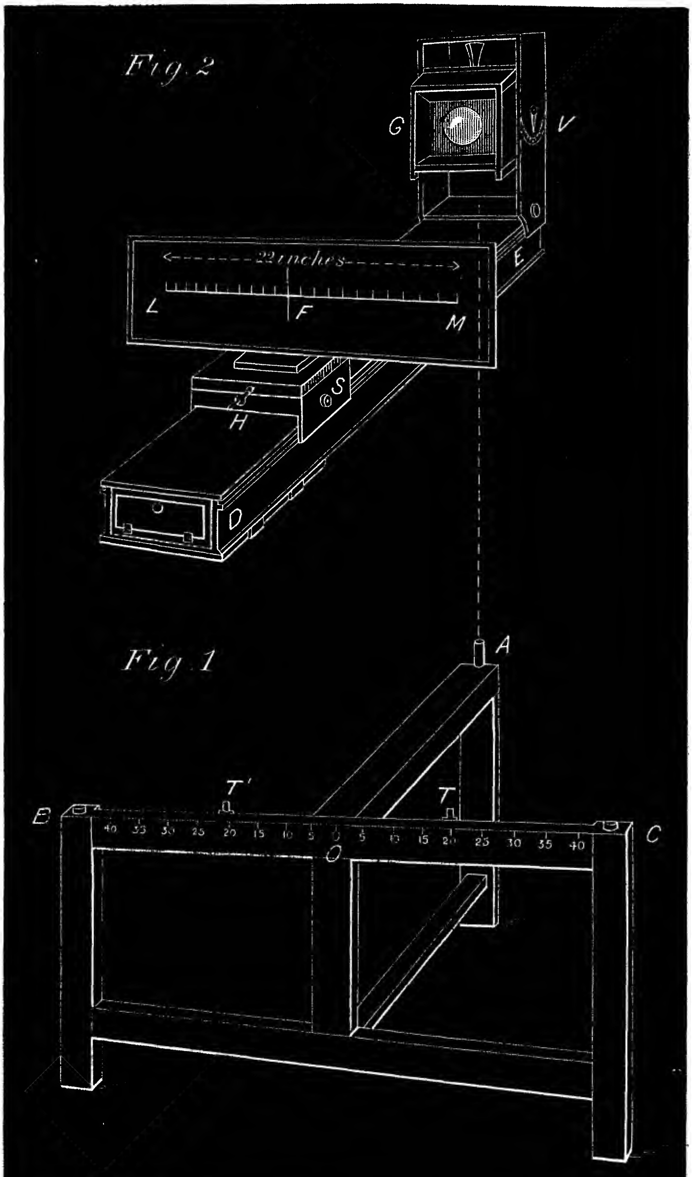
The defect known as *flare spot* consists of a bright spot or patch of light being formed in the centre of the field. To detect it, the lens is

placed in an ordinary camera, which should be pointed at the sky; if the ground glass is brought to the principal focus, the flare spot is then readily visible.

For tests Nos. 9 to 16 an apparatus designed by myself, and which I have called the "testing camera," is used. It is neither an expensive nor an elaborate contrivance, and there can be no doubt that if more money had been expended a more perfect machine could have been made. Until a system of this sort has been in regular use for some time, and until it has stood the fire of criticism, experience shows, we think, that the apparatus employed is apt to be little more than a good working model of what it will become by future developments; but improvements would in this instance probably tend to increased rapidity rather than to increased accuracy, for the results obtained are now quite accurate enough for all practical purposes. Even now alterations are under consideration, such as the substitution of a sliding eye-piece on a graduated bar for the long sheet of ground glass. For the above-mentioned reasons, and because much expenditure could not be justified until it was certain that lenses would be sent for examination in considerable numbers, the Kew Committee raised no objection to the somewhat make-shift appearance of the apparatus.

The general idea of the testing camera is extremely simple, but the name perhaps is hardly a happy one, as there is no "camera" or chamber about it. Except for the absence of bellows, it may be said to consist of the essentials of an ordinary camera, which is capable of being revolved horizontally about a vertical axis passing through the lens; though it must be confessed that this description gives no idea of its appearance. The three-legged stool or bench, seen in fig. 1, represents the legs of the camera, and fig. 2 shows the apparatus that takes the place of the body; G is the lens-holder, and LM the ground glass, both of which are capable of independent movement backwards and forwards on the hollow wooden beam DE, called the "swinging beam." There is a conical brass cap or pivot, not shown in the sketch, under the upper plank of the swinging beam, underneath where the lens-holder G is shown in the sketch. The whole of the apparatus shown in fig. 2 is placed on the top of the three-legged stool, the round-headed iron pin A passing loosely through a hole in the lower plank of the swinging beam, and fitting into the conical brass cap or pivot. The swinging beam, being thus supported by the pin A and by the long arm BC of the stool, is capable of being revolved round A as a centre. On the ground glass is engraved a horizontal line, which is accurately divided into fiftieths of an inch; this line passes through the centre of the ground glass (or through the point where the perpendicular from the lens-holder cuts the glass), and is also parallel to BC, the top of the stool on

FIGS. 1 AND 2.



which the swinging beam slides, when the camera is in position ; thus the image of an object will appear to run along the scale as the swing-

ing bar is moved from side to side. The ground glass can be brought approximately into focus by means of the already-mentioned movement to and fro on the swinging beam, but for accurate adjustment a slow motion arrangement is attached to the movable part itself; the handle H gives the required motion, and there is a scale S, called the "focus scale," by means of which these small movements can be accurately measured. On the lens-holder there is a movement, corresponding to the swingback of an ordinary camera, by which the lens can be made to revolve vertically round a horizontal axis, without, of course, any corresponding movement of the ground glass; there is a vertical arc, V, by means of which we can read off the vertical angles through which the lens is rotated. An arrangement is also suppled by means of which the lens can be moved backwards and forwards on the movable stand, thus allowing the position of the lens to be so adjusted that the horizontal axis can be made to pass through any point in its axis.

9. *Effective Aperture of Stops.*

Number engraved on stop.	Effective aperture. Inches.	*f*/number.	C.I. No.
No.........	....	....	....
No.........	....	....	....
No.........	....	....	....
No.........	....	....	....
No.........	....	....	....
No.........	....	....	....
No.........	....	....	....

The effective aperture of one or more of the various stops supplied with the lens is found by a well-known method. The image of a very distant object is first brought into focus on the ground glass of the testing camera; a collimator, which has itself been previously focussed on a distant object, may be used instead of the distant object; the ground glass is then taken out and exactly replaced by a tin plate with a small hole at the centre; this hole, which should be very small, will, therefore, be at the principal focus of the lens. The room being darkened, a gas burner is placed behind the small hole, and thus parallel rays, in the form of a cylinder, are made to issue from the lens towards the front. A piece of ground glass, with a graduated scale engraved on it, is now held in front of the lens, and the diameter of the illuminated disc, or section of the cylinder as seen on the glass, is directly measured off as any stop is inserted in its place. Thus is found the

effective aperture of the largest stop, as recorded in the Kew Certificate of Examination. The ratio of the effective aperture to the diameter is the same for all stops of the same lens, and the effective aperture of the other stops is either measured as above, or calculated from the ratio thus found. As the rays are parallel when emerging from the lens, it is evident that, if the stop is in front of all the lenses, the effective aperture will be the same as the diameter of the stop itself.

By imagining the path of the rays in the above experiment as being reversed, in which case the rays forming the cylinder are all brought to a focus on the plate, it is evident that the intensity of illumination of the plate at the centre, when focussed for distant objects, varies directly as the sectional area of the cylinder, and therefore as the square of the effective aperture as above measured. The intensity of illumination of the plate also varies inversely as the square of the distance from the point in the lens from which all the light may be supposed to emanate, that is, from the nodal point of emergence. If we adopt as our definition of the principal focal length (f) of the lens the length between the principal focus and the nodal point of emergence, it is then evident that the square of the effective aperture divided by f^2 will be a measure of the illumination of the plate. In consequence of this fact, it has for a long time been customary to speak of the diameter of stops in terms of the ratio of their effective apertures to the focal length of the lens; for example, a lens having a stop with an effective aperture equal to one-tenth of its principal focal length is commonly spoken of as working with an intensity of $f/10$. But it has recently been found by photographers that it is practically useful to adopt a definite standard or unit of intensity of illumination in order that the different stops may be numbered in such a way as to readily indicate the different exposures required with each; many systems of this kind have been considered: $f/100$, $f/10$, $f/4$, and $f/\sqrt{10}$, each having been at various times proposed as the basis of enumeration, the numbering of the stops sometimes increasing and sometimes diminishing as the necessary exposure increases. Each of these systems has met with considerable opposition from different quarters; but this is not the place to discuss their relative merits, more especially as in selecting one of them for the Kew certificates, we have been chiefly influenced by considering which has received the sanction of the most authoritative body, and which, therefore, stands the best chance of universal adoption. Judged by this standard, there can be no doubt that the recomendations of the International Photographic Congress of Paris of 1889, as endorsed by the Congress at Brussels, should be adhered to as far as possible.

The following system, which we have called the C.I. system, has therefore been adopted at Kew. The stop $f/10$, the effective aperture

of which is one-tenth of the principal focal length of the lens, is called stop No. 1; and the exposure necessary for any subject with that stop is the unit of exposure for that subject. The other stops are numbered in the inverse ratio of the area of their effective apertures to the area of the effective aperture of stop No. 1. Thus stop No. 2 gives half the intensity of illumination of stop No. 1; and, in any case, to find the time of exposure necessary to produce the same result as with the unit of exposure with stop No. 1, we multiply that unit by the number of the stop in use. The practical rule to find the C.I. number of a stop is to divide the square of the principal focal length by 100 times the square of the diameter of the effective aperture of the stop. The principal focal length, which we require to know in order to calculate the numbering of the stops, is found by test No. 11.

The difficulty of introducing the C.I. numbering of stops will perhaps be greater in England than on the Continent, partly because, previous to the Paris Congress, the Photographic Society of Great Britain had given provisional support to another system based on $f/4$ as a unit. The Photographic Society has been waiting for the recently published reports of the Brussels Congress to reconsider this matter, and it may be hoped that they will join in the effort to get the C.I. system universally adopted, notwithstanding the inconvenience that must be severely felt at first by those who are therefore obliged to change their methods.

10. *Angle of Cone of Illumination with Largest Stop = —— °, giving a Circular Image on the Plate of —— inches diameter. Angle of Cone outside which the Aperture begins to be Eclipsed with Stop C.I. No. —— = —— °, giving a Circle on the Plate of —— inches diameter.*

Diagonal of Plate = —— inches, requiring a Field of ——° (= 2ϕ).

Stop C.I. No. —— is the Largest Stop the whole of the Opening in which can be seen from the whole of the Plate.

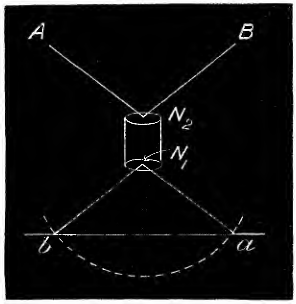
If a stop, or thin metal diaphragm with a circular aperture, is revolved round any axis passing through its plane, and if it is regarded from a little distance, the whole aperture, foreshortened of course, can be seen except in one position in each half revolution; if in a similar way a piece of tubing is revolved about an axis at right angles to its own axis, there is only one position in which the whole of the aperture can be seen, and any movement from this one position will cause the opening to begin to be eclipsed, thus giving it a lozenge-shaped appearance; as the movement goes on, this opening will get smaller and smaller till it is quite obliterated. In looking through a lens as it is revolved about an

axis perpendicular to its own axis, it will be seen that, as a rule, something between these two extremes occurs; commencing from a position when we are looking directly along the axis, no other result than foreshortening the opening is at first produced by the revolution of the lens; then comes an angle at which the aperture in the stop begins to be eclipsed, either by the mounting of the lenses, or by fixed diaphragms, &c.; lastly, we come to an angle at which the lozenge-shaped opening appears to vanish, and no light is seen to come from the lens. It is obvious that the intensity of illumination of different parts of the photographic plate varies with the size of the aperture visible from each point; and, neglecting other considerations for the present, there is thus an inner cone, forming a disc where it cuts the plate, in which the illumination decreases regularly from the centre outwards according to a known law: and there is an outer cone, forming an annulus between where it and the inner cone cut the plate, in which the illumination decreases more rapidly than according to the above-mentioned law; very rapidly, therefore, probably irregularly, on account of the aperture of the stop being successively eclipsed by different parts of the mounting, and certainly according to no law that can be readily stated or ascertained. The test now under consideration gives the angles of these two cones.

The outer cone, which we have called the "cone of illumination," gives the extreme angle of the field of the lens without regard to definition, and is what is known to French authors as the *champ de visibilité*. To find the angle of the cone of illumination, the lens is placed in the testing camera, and the observer looks through the small hole in a sheet of tin plate with which the ground glass has been replaced, as in the last test; the lens-holder is made to revolve about its horizontal axis, and as the axis of the lens moves away from zero, first in one direction and then in the other, the positions at which all light appears to be cut off are noted; the angle between these two positions as read on the vertical arc, V, gives the angle of the cone of illumination.

In order to ensure correct results it is necessary that the axis of rotation should pass through the nodal point of emergence. If in fig. 3 AN_2N_1a and BN_2N_1b represent the extreme rays forming the cone, N_2 and N_1 being the nodal points, it is evident that in order to measure the angle bN_1a of the cone the lens must be revolved about N_1, the nodal point of emergence, as a centre. The necessary adjustment is made in the following manner:—The image of a distant object having been thrown on the ground glass, the lens is turned through a small angle about the horizontal axis, the glass remaining stationary. If the movement of the lens gives rise to any movement in the image, then the axis does not pass through the nodal point of emergence

FIG. 3.



and an adjustment is necessary; this is done by moving the lens-holder in or out, thus making the axis of rotation pass through different parts of the axis of the lens, until the image ceases to show any movement; and this can only be the case when the axis of rotation does pass through the nodal point of emergence. As far as the above considerations are concerned, it is immaterial how far off the small hole in the tin plate is from the lens, but if the horizontal axis has not been made to pass accurately through the nodal point of emergence, this want of adjustment will have much the same effect as a small vertical movement between the two readings of the vertical arc. It is evident that the angular error thus produced will diminish as the distance of the point of observation increases; moreover, any distortion at the edge of the plate will make the above theoretical considerations no longer strictly applicable, and will have the same effect as the axis of rotation not accurately passing through the nodal point. In order, therefore, to minimise these sources of error, the tin plate with the hole in it is removed as far as practicable from the lens before the observation is made.

The angle of the inner cone, that is, of the cone outside which the opening of the stop is partially eclipsed by the mounting of the lens, &c., is measured in the same way as above described for the outer cone, and with the same precautions. When looking through the small hole, the positions on each side of zero at which the aperture begins to be shut off, and beyond which it no longer appears as a perfect ellipse, are easily seen, and the angle between these two positions as measured on the vertical arc gives the angle required. The angles of these two cones are generally given when the observation is made with the largest stop supplied with the lens.

The results of these measurements should be considered in connexion with test No. 17, under which heading the general question of the illumination of the field will be discussed. In order to facilitate the consideration of the covering power of the lens, the diameters of

the circles which these cones make by cutting the photographic plate, when the focus is adjusted for distant objects, are given in the Certificate of Examination. Having found the principal focal length in the manner to be described immediately, the size of these circles can readily be ascertained by a simple graphical method, which is hardly worth describing in detail.

In connexion with this test it may be convenient to adopt the use of the term *angle of field under examination*, (denoted in this paper by 2ϕ,) to signify the angle subtended at the nodal point of emergence by a diagonal of the plate, or the greatest angular distance which could be included in the photograph, supposing the focus to be taken on a distant object. This angle is found by the graphical method mentioned above for determining the diameter of the circles on the plate, and the result is entered on the Certificate of Examination.

If the illumination of the field is not to fall off rapidly towards the edges of the plate, for the normal use of the lens we should employ a stop which covers (or nearly covers) the plate of the given size with its inner cone; that is to say, we should use a stop not larger than the largest stop the whole of the opening in which can be seen from the whole of the plate. In order to find the largest stop which fulfils the above conditions, the lens is revolved about the horizontal axis until the vertical arc reads half the angle of field under examination, and then the different stops are put in one by one until the largest one is found which is seen not to be eclipsed when the observation is made through the hole in the tin plate. The number of this stop is recorded in the certificate.

The readings taken when measuring the angles of these cones are also utilised for the purpose of adjusting the position of the lens in a manner necessary to ensure accuracy in several of the following tests. The vertical arc is so arranged that it reads zero when the axis of the lens is horizontal, that is to say, when the axis passes through the small hole in the tin plate from which the observation is made; hence the two readings on the arc when the lens is revolved about the vertical axis, first one way and then the other, so as just to cut off all the transmitted light, should be exactly the same; if they are not identical the lens-holder is placed in such a position that the reading on the vertical arc is equal to half the difference between them; then it is evident that the mechanical axis of the objective passes through the small hole, or at all events, cuts the tin plate on the same level as the hole. Now this small hole in the tin plate is in the same position as the centre of the engraved line when the ground glass is in position. Hence, this adjustment being made, in future tests we may consider that the mechanical axis of the lens cuts the line on the ground glass near its centre.

11. *Principal focal length* = —— *ins.*
Back focus, or length from the principal focus to the nearest point on the surface of the lenses = —— *ins.*

The following is the method of finding the principal focal length with the testing camera. By means of the mark 0 (see fig. 1), on the three-legged stool, the swinging beam can be brought approximately to a central position; there are also two iron stops, T and T′, removable when not wanted, which, when in position, prevent the swinging beam from passing beyond these points. These stops (or, more accurately, the iron plates on the swinging beam with which they come in contact) are capable of adjustment, and thus a means is obtained of allowing the beam to be revolved about A as a centre, through a known angle, with great ease and accuracy. After the focus has been very carefully adjusted for a distant object, and after the beam has been brought approximately to the central position by means of the mark 0 on the stool, the image either of some well-defined object seen through a hole in the window-shutters, or of a mark in the collimating telescope, is made to appear on the centre of the engraved line on the ground glass; this can be done by raising or lowering one or more of the legs of the stool or by moving it laterally; this adjustment being accurately made, the line joining F, the centre of the ground glass, and the centre of the lens, if prolonged, will pass through the distant mark; when once made, this adjustment will hold good, with sufficient accuracy, for all lenses which may subsequently be placed in the testing camera. Now, when the swinging beam is moved from side to side, the image appears to run along the engraved line on the ground glass; the position of the image is first noted when the beam is in contact with the stop T, and afterwards when in contact with the stop T′; twice the distance, as measured on the scale, between these two points gives the principal focal length of the lens under examination.

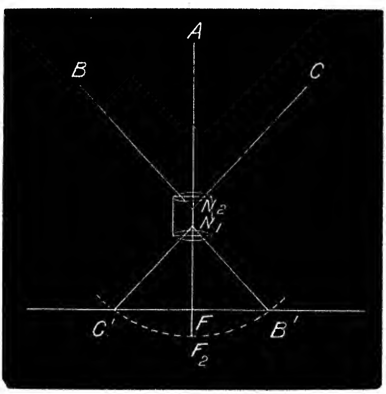
In order to ensure accuracy, certain precautions must be taken. The object must be so far off that the distance between its focus and the focus of a point in the same direction at an infinite distance is considerably less than the probable error of observation. The chief difficulty of finding the principal focal length, in the Kew method, and, indeed, in all methods, consists in obtaining an accurate adjustment for focus; and since, for a given error in focus, the greater the aperture the more diffusion there is in the image, the largest stop should always be used when focussing; but there is no objection to slipping in a smaller stop after the focus is taken so as to obtain as sharp an image as possible, and thus make it easier to read the position on the scale with accuracy.

Before proving that the result above obtained is, in fact, the prin-

cipal focal length of the lens, it may be as well to give a rigid definition of what is here meant by that expression, as it has often been used in somewhat different significations. The definition here adopted of the principal focal length is the distance between the principal focus for visual rays (or the image as seen by the eye of an infinitely distant point on the axis of the lens) and the nodal point of emergence. The use of the term *nodal point* is, perhaps, open to criticism; under the ordinary circumstances of a photographic lens the nodal points and the principal points occupy the same positions, and therefore either of these expressions might have been used in the definition; but if we take into consideration any imaginary circumstances when these two points would not be identical, as, for instance, if one end of the lens was immersed in water, it will be observed that the Kew method of determining the principal focal length would find the distance between the nodal point and the ground glass, and not that from the principal point; moreover, under these imaginary conditions, it would be the distance of the nodal point from the plate which would chiefly be of value to the aquatic photographer, for the intensity of illumination of his plate would vary as the square of that distance and not of the distance from the principal point. But it must be confessed that the term was, in reality, adopted because it is that best known in the photographic world, and not on account of such hair-splitting reasons as these.

It now remains to be seen if the Kew method does give the true principal focal length according to the above definition. In fig. 4, let

FIG. 4.



B, A, and C be three very distant points, A being on the axis of the lens, and B and C being at equal angular distances on either side of it; let N_1 and N_2 be the nodal points; let C′, F, and B′ be the images

of these three points on the ground glass, when, if the distance N_2A is great enough, F will not be further from the principal focus than the error of observation, and may, therefore, be confounded with it. The angle, BN_2C, subtended by the points B and C at the lens, can easily be measured, and, since the incident and emergent rays passing through the nodal points are parallel to each other, the angle $C'N_1B'$ is thus obtained; the distance, $C'B'$, that is, the distance between the images of the two outside points, can be also measured on the ground glass: $C'B'$ and $C'N_1B'$ being given, FN_1 can therefore be found; for since, by supposition, the line AN_2 bisects the angle BN_2C, FN_1 is equal to $C'B'/2 \cot C'N_1B'/2$. This, therefore, is a method by which the principal focal length, as defined above, can be measured. But if, instead of having objects at known angles, only one object is observed, and the camera is revolved round the point N_1, through the angle $C'N_1B'$ between the observations, exactly the same result can be obtained; this is the method adopted at Kew. The movement in parallax of the point N_2 is so small that it may be neglected. The advantage of this method is that a collimating telescope can be used as the object, and thus, during dull weather, the work can be carried on indoors. In working with the testing camera, the angle $C'N_1B'$ represents the angle through which the swinging beam is revolved about the vertical pivot; the stops are arranged so that $C'N_1F = \tan^{-1} \frac{1}{4}$, that is, so that $C'B' = 2\ FN_1$; and, therefore, twice the distance $C'B'$ measured on the ground glass gives FN_1, the principal focal length of the lens. The Kew method, therefore, gives the result required.

It might at first sight appear that a considerable error would be due to the fact that the adjustment to the central position is merely made by a rough mark, and that it is only the total angle $C'N_1B'$ (that is, the angle moved by the swinging beam between the iron stops) which is accurately known. It is true that it can only be said that $C'N_1F$ is approximately equal to FN_1B'; but if $C'N_1B'$ is less than 90°, and if the line N_1F does not differ in direction from the true central position by more than 1°, then the principal focal length obtained in this manner does not differ from the truth, for this reason, by more than 1/17th per cent. As it is considered that this would represent an extreme case, it is therefore evident that this is a negligible source of error.

In order that the Kew method of finding the principal focal length should not be open to any criticism on theoretical grounds, three conditions must be fulfilled: it is obvious that these conditions need not hold good further from the axis of the lens than the points at which the observations were made. 1st. The principal focal surface, or the locus of the focus for very distant objects, must be a plane. 2nd. The image must not be distorted. 3rd. The nodal point of

emergence for visual rays should be the same as the nodal point for actinic rays.

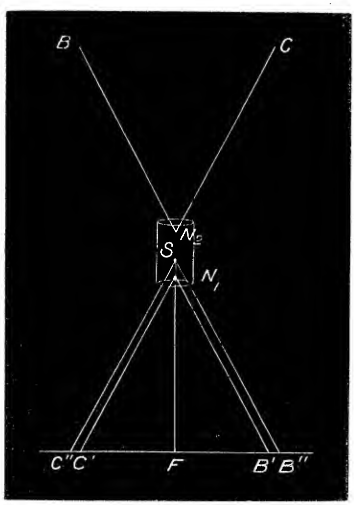
In no lens are these conditions perfectly fulfilled, but before discussing the nature of the errors thus introduced it may be as well to consider shortly for what purposes and with what degree of accuracy the practical photographer wants to know the focal length of his lens. Two uses to which this knowledge would or could be put have already been mentioned, and we know of no others. In the first place, it has been shown how the numbering of the stops depends on the focal length, and how advantageous is the knowledge of the intensity of the illumination of the plate which may thus be gained. But as, on account of the difference in the amount of reflection and absorption of the lenses, two lenses with the same C.I. number of stop may differ more than 10 per cent. in the intensity of illumination in the centre of the field, as in the same objective the difference of illumination of different parts of the field is generally more than 20 per cent., and as the photographer is seldom able to estimate his unit of exposure within this latter percentage, it can hardly be seriously contended that the focal length must be known with very great accuracy for this purpose. The second object for which the photographer may require to know the focal length is for the use of the tables in which the distance is given at which the object has to be placed to obtain a given enlargement or reduction; it has already been stated that this is not, we believe, a want often felt, except for getting approximate results; but if the focal length is used for final adjustments in this manner, it should be known with very considerable accuracy.

With regard to the first condition, as to the focal surface being a plane, it should first be stated that it is found convenient at Kew to bring the ground glass into focus when the swinging beam is in contact with one of the stops, thus insuring the greatest sharpness of image at the points of observation; that is to say, in fig. 4, the principal focal surface is made to pass through the points B' and C', and, if it is not a plane, it may be represented by the dotted curve $C'F_2B'$. Under these circumstances, therefore, the principal focus will be at F_2, and N_1F_2 will represent the principal focal length according to our definition; but it has been shown that the observation gives N_1F as the focal length, thus introducing an error equal to FF_2 in the result. It is to be observed, however, that with a lens giving a markedly curved focal surface, the photographer, in order to get a general minimum amount of diffusion, would adjust his focus by looking at the image at a point somewhat more than half way from the centre to the margin of his plate: for example, with a lens covering 50° or 60°, he would focus at a point some 15° from the centre, or at about the position where the Kew observation for the

focal length is taken; thus, with such a lens, C′B′, in fig. 4, would represent the position of the photographic plate; and it is evident that, for all questions of illumination or enlargement, N_1F, or the distance from the plate to the point from which all the light may be supposed to emanate, should be introduced into the calculations, and will give the true results, or, at all events, more nearly the truth than if N_1F_2, the true principal focal length, had been used in its place. Thus, by recording the length N_1F in the Certificate of Examination, we always give more nearly what the photographer practically wants than if the length N_1F_2, or the true principal focal length, had been ascertained. But in any case the point raised in this paragraph could, if thought desirable, be met by focussing the plate in the centre of the field when the observation for focal length is made.

The second point raised, as to the theoretical correctness of the principal focal length as found at Kew, is with regard to the distortion of the image, which may be described as the results due to the theory of the nodal points being not strictly applicable except near

Fig. 5.



the centre of the field. In fig. 5, let N_1 and N_2 be the nodal points, F the principal focus, and B″ and C″ the images of the infinitely distant points B and C; if there is distortion, the lines SB″ and SC″, drawn parallel to the incident rays, do not cut the axis at N_1, the nodal point of emergence; let these lines cut each other at S, which may be called the principal point of similitude with regard to the images B″ and C″. This construction represents the Kew method of observation, and therefore SF is the distance found as the principal

focal length, thus introducing an error equal to SN_1 in the result; the focal length given is, in fact, the distance from the principal focus to the principal centre of similitude for the part of the plate where the observation is made. But here again, since BN_2C, the cone of incident rays, is spread over a disc on the plate of which $B''C''$ is the diameter (and not $B'C'$), the mean intensity of illumination of the plate between these points will vary inversely as $(SF)^2$; and, if the plate covers an angle larger than BN_2C, the C.I. numbering of the stops will give a better indication of the relative exposure on the assumption that SF is the principal focal length than if the true value N_1F is introduced into the calculations. Thus, what has been given in the Certificate of Examination will again be nearer what is practically required by the photographer than if the true principal focal length has been recorded. If, however, the lens is intended to be used for enlargements or reductions, and the final adjustment of the distance of the object is to be made by reference to tables, then, no doubt, the true principal focal length must be accurately given; but no photographer would ever use a lens showing sensible distortion within 15° of the axis, for such purposes, for, if he did, the ratio of the enlargement or reduction would vary sensibly in different parts of his plate; and, if there is no distortion within this distance from the axis, S and N_1 will be coincident, and the Kew method will give accurately and truly N_1F as the principal focal length. Thus, in the only circumstances under which the principal focal length is practically wanted with theoretical truth and great accuracy, it is seen that the results given in the Kew certificate do answer these requirements.

The third condition that has been laid down as being necessary before the Kew method gives theoretically correct results is that the nodal point should be the same for white light as for photographically actinic rays. This may be hypercritical, but if, in fig. 5, C'' and B'' represent the images as seen on the photographic plate, C' and B' those seen by the eye on the ground glass, N_1 the mean position of the nodal point of emergence for visible rays, and S the mean position for actinic rays, then it is evident that FN_1 will be the principal focal length found by the observation, whereas SF will be the quantity required in calculations with regard to enlargements or illumination. If the lens gives any distortion, N_1 would represent the centre of similitude for visible rays and S that for photographically actinic rays; the condition might, therefore, have been more rigidly defined by stating that the point of similitude for visible rays and that for actinic rays must occupy identical positions for parts of the field between the points of observation. As far as can be judged, this is a negligible source of error in all cases.

A fairly large angular movement of the swinging beam, about

$14\frac{1}{2}°$ on each side of the axis, has been adopted at Kew in order that any error in the measurements on the ground glass may produce a small proportional error in the results. But it should be observed that, the smaller this angle, the less will be the errors just discussed, and by lessening the angular movement these errors can be reduced to any extent, but only with a proportional loss in the general accuracy of the results obtained.

This is not the place to enter into a general discussion on focometry, but a few words to justify the choice of the Kew method may perhaps be permitted. Many of the known means of finding the principal focal length depend in principle on measuring the relative size of the object and the image, and the foregoing remarks on the errors involved are more or less applicable to them, thus showing that they are open to the same criticisms on theoretical grounds as the work at Kew. Many methods of focometry have to be rejected because they do not measure the distance from the nodal point, and others are unsuitable because the calculations or successive adjustments involved, render the operation too lengthy. There are no doubt many instruments, as for instance that devised by Professor Silvanus Thompson, which do give the true focal length as measured on the axis with theoretical accuracy, but these have not, as a rule, been specially designed for photographic lenses. One method, which is hardly open to criticism on theoretical grounds, may be mentioned in a little greater detail as being that specially recommended by the International Congress of Paris; this is the elegant plan which Commandant Moëssard proposes to carry out by means of his instrument, called the *Tourniquet*, which is described in Wallon's 'Traité élémentaire de l'Objectif Photographique,' and elsewhere. Advantage is taken of the principle that if a lens is revolved about an axis passing through the nodal point of emergence, the image of a distant point will not appear to move if seen through a fixed eye-piece; thus, by successive adjustments and trials, the lens can be so placed that an axis does pass through the nodal point; and, by measuring the distance between this axis and the focus of the eye-piece, the true focal length can therefore be obtained. Since a movement can be detected before it can be measured, a smaller angular movement is required with this method than with the Kew testing camera, and therefore, as far as distortion is concerned, greater, but not absolute, theoretical accuracy is obtained. As for the coincidence of the visual and actinic centres of similitude, better theoretical results are only obtained by this method on the assumption, which is probably a true one, that these points approach each other as the point of observation gets nearer the axis.

By taking observations some 14° away from the axis of the lens, we conclude, therefore, that we obtain the most rapid and

accurate method of focometry, and, in the case of the image within this limit being distorted, that the focal length thus obtained, even though it is not identical with the principal focal length measured on the axis, is what the photographer in reality wants to ascertain. The Kew method is, therefore, we believe, open to no criticism on theoretical grounds as far as the value of the results is concerned.

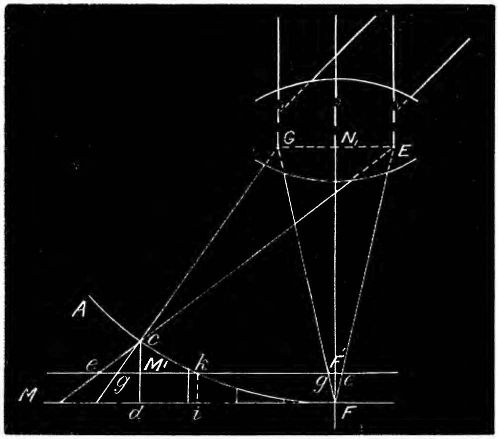
Under the same heading as the principal focal length is also recorded the "back focus," or the length, from the ground glass surface to the nearest summit of the lenses, when the focus is adjusted on a distant object. The difference between the principal focal length and the back focus therefore gives the distance of the nodal point of emergence from the inner summit of the lenses, thus enabling anyone to mark the place where the principal plane cuts the mounting. In symmetrical lenses, which are generally used for plan work, the position of the principal plane of incidence, or the point from which the distance of the object must be measured when regulating enlargements, can also be marked; for it then occupies the same relative position with regard to the furthest summit of the lenses (that is, to the outside end of the lens) as the nodal point of emergence does to the inner summit of the lenses.

12. *Curvature of the Field, or of the Principal Focal Surface. After focussing the plate as its centre, movement necessary to bring it into focus for an image —— inches from its centre = —— inches.*
Ditto for an object —— inches from its centre = —— inches.
" " —— " " = —— "

The following is the method of finding the curvature of the principal focal surface. The image of a distant object (or of the collimating telescope) is thrown on that point on the ground glass where the axis of the lens cuts it, the focus is accurately adjusted, and the focus scale is read off. The swinging beam is then moved so that the image comes successively to positions at convenient intervals from the centre of the plate, and on each occasion the focus is adjusted afresh, and the focus scale read off. By subtracting the central reading from these outer readings, the results recorded in the Certificate of Examination are obtained.

But a mere observation of the curvature of the focal surface does not at once indicate how serious is the evil effect of this defect in the lens. Further consideration is necessary to settle this point. If the results furnished by this test are plotted in the form of a curve, they will represent a section through the principal focal surface; let AF in fig. 6 be such a curve, and let cd $(= \mu)$ be the movement necessary to bring the plate into focus at its margin; let N_1F be the principal focal length, and $EG (= \epsilon)$ the effective aperture of the

FIG. 6.



lens. The effect of this curvature is to make the image of a point appear on the plate as a disc, except on the circle or at the point where the principal focal surface either cuts or touches the plate. If the photographic plate is in the position M′F′, such that it bisects *cd*, then the discs of diffusion will be greatest at the centre and at the margin of the plate; and any movement of the plate from this position will increase the size of the disc at one or other of these places; if the photographer adjusts his focus so as to produce the best general focus, *eg* (= δ) will therefore be the diameter of the largest disc of diffusion on his plate. Assuming that this position of the plate has been adopted, and that the lens gives no distortion, then, by similar triangles, it can be seen that:—

$$\mu = \frac{2\,\delta}{\epsilon}(f-\mu) = \frac{2\,\delta f}{\epsilon} \text{ nearly} \ldots\ldots\ldots\ldots \quad (1).$$

But the C.I. No. of stop $= \dfrac{f_2}{100\,\epsilon^2}$; and therefore

$$\mu = 20\,\delta\,\sqrt{(\text{C.I. No. of stop})} \ \ldots\ldots\ldots\ldots \quad (2).$$

The following table gives the value of δ for different values of μ, and for stops of different numbers; and thus the size of the greatest disc of diffusion can at once be seen from the results of the examination as recorded in the certificate.

Table giving μ (the difference of focus in inches of the centre and the margin of the plate; or, after focussing the plate at its centre, the amount it has to be moved to bring the margin into focus) with reference to the size of disc of diffusion and number of stop.

C.I. No. of stop.	δ, the diameter, in decimals of an inch, of the maximum disc of diffusion when the plate is in the position giving the best general focus.									Approximate ratio of effective aperture to focal length.
	0·002.	0·004.	0·006.	0·008.	0·010.	0·012.	0·014.	0·016.	0·018.	
64	0·32	0·64	0·96	1·28	1·60	1·92	2·24	2·56	2·88	*f*/80
60	0·31	0·62	0·93	1·24	1·55	1·86	2·17	2·48	2·79	*f*/77
56	0·30	0·60	0·90	1·20	1·50	1·80	2·09	2·39	2·69	*f*/75
50	0·28	0·57	0·85	1·13	1·41	1·70	1·98	2·26	2·55	*f*/71
48	0·28	0·55	0·83	1·11	1·39	1·66	1·94	2·22	2·49	*f*/69
40	0·25	0·51	0·76	1·01	1·27	1·52	1·77	2·03	2·28	*f*/63
32	0·23	0·45	0·68	0·91	1·13	1·36	1·58	1·81	2·04	*f*/57
30	0·22	0·44	0·66	0·88	1·10	1·32	1·53	1·75	1·97	*f*/55
24	0·20	0·39	0·59	0·78	0·98	1·18	1·37	1·57	1·76	*f*/49
20	0·18	0·36	0·54	0·72	0·89	1·07	1·25	1·43	1·61	*f*/45
16	0·16	0·32	0·48	0·64	0·80	0·96	1·12	1·28	1·44	*f*/40
15	0·15	0·31	0·46	0·62	0·77	0·93	1·08	1·24	1·39	*f*/39
12	0·14	0·28	0·42	0·55	0·70	0·83	0·97	1·11	1·25	*f*/35
10	0·13	0·25	0·38	0·51	0·63	0·76	0·88	1·01	1·14	*f*/32
8	0·11	0·23	0·34	0·45	0·57	0·68	0·79	0·91	1·02	*f*/28
6	0·10	0·20	0·29	0·39	0·49	0·59	0·69	0·78	0·88	*f*/24
5	0·09	0·18	0·27	0·36	0·45	0·54	0·63	0·72	0·81	*f*/22
4	0·08	0·16	0·24	0·32	0·40	0·48	0·56	0·64	0·72	*f*/20
3	0·07	0·14	0·21	0·28	0·35	0·42	0·48	0·55	0·62	*f*/17
2	0·06	0·11	0·17	0·23	0·28	0·34	0·39	0·45	0·51	*f*/14
1	0·04	0·08	0·12	0·16	0·20	0·24	0·28	0·32	0·36	*f*/10
$\frac{3}{4}$	0·03	0·07	0·10	0·14	0·17	0·21	0·24	0·28	0·31	*f*/8·7
$\frac{2}{3}$	0·03	0·07	0·10	0·14	0·16	0·20	0·23	0·26	0·29	*f*/8·2
$\frac{1}{2}$	0·03	0·06	0·08	0·11	0·14	0·17	0·20	0·23	0·25	*f*/7·1
$\frac{2}{5}$	0·03	0·05	0·08	0·10	0·13	0·15	0·18	0·20	0·23	*f*/6·3
$\frac{1}{3}$	0·02	0·05	0·07	0·09	0·12	0·14	0·16	0·18	0·21	*f*/5·8
$\frac{2}{7}$	0·02	0·04	0·06	0·09	0·11	0·13	0·15	0·17	0·19	*f*/5·3
$\frac{1}{4}$	0·02	0·04	0·06	0·08	0·10	0·12	0·14	0·16	0·18	*f*/5·0

When judging the quality of a lens by means of the results given in this test, the above table may also be used in the following manner:—Decide on the value of δ (the diameter of the greatest disc that will be tolerated in the image of a point), and find, from the results recorded in the Certificate of Examination, the difference of focus, μ, between the centre and the extreme corner of plate; then, knowing these two quantities, the table at once shows what is the C.I. number of the stop that can be employed under these conditions, or, in other words, with what rapidity the lens will work.

It may also be remarked that this table gives for any part of the plate, and for stops of given size, the *radius* (δ) of the image of a point after the plate has been removed a distance, μ, from its proper focus in either direction, the movement being measured in a direction perpendicular to the plane of the plate.

According to the recommendations of the International Congress, lenses should generally be supplied with stops, numbered according to the proposed system, in the following series: 1, 2, 4, 8, 16, 32, 64, &c. I should have thought that the series, 1, 2, 3, 5, 10, 20, 30, 60, &c., would have been more convenient for the purposes of mental arithmetic; for example, with the two last stops in this series, the exposure would be the same multiple of the half minute or minute that the unit of exposure is of the second. Both series have therefore been included in the above table.

The results recorded in the certificate under this heading may possibly also be useful to the photographer in another way, by enabling him to decide approximately what part of the ground glass he should use when focussing. In fig. 6, let k be the point where the plate M′F′ cuts the principal focal surface when in the position which has been proved to give the best general focus; hence the image will be perfectly sharp at k, and conversely, if the focus is adjusted by looking at the point k on the ground glass, the plate will be brought to the position M′F′ required; but since kl is half cd, there is no difficulty by interpolation or plotting to find the approximate position of k for any given distance of the point c from the axis. Look in the Certificate of Examination for cd, the difference of focus between the centre of the plate and its margin, find in the above manner the position of k, where the difference of focus from the centre is only half cd, and we get the point on the ground glass which should always be used when focussing with all stops, if it is desired to get the best general focus.

13. *Definition at the Centre with the Largest Stop, —— C.I. Stop, No. —— gives —— definition over the whole of a —— inch by —— inch plate.*

The system by which the defining power is measured consists in ascertaining what is the thinnest black line of which the image is just visible, the test being conducted in the following manner. The test object consists of a thin straight strip of steel, about 0·1 inch wide, and about an inch long; it is capable of being rotated about an axis in the direction of its greatest length, thus, if seen against a bright background, making it appear as a black line of varying width; when presented edgewise to the objective, it is so thin that the image becomes invisible; and there is an arc so graduated that the angle subtended by the two edges of the strip at the lens can be at once read off, thus giving a measure of the apparent thickness of the line. The test-object is placed as far as possible from the lens in a darkened room (at Kew the accommodation in this respect leaves much to be desired), and beyond it is a ground glass screen illuminated by a lamp.

In order to test the defining power of a lens in the centre of its field, the focus is first very carefully adjusted on the ground glass, and the test-object is then slowly revolved from the edgewise position, where its image is invisible, until the first appearance of a dark line can be seen against the bright background; the angular width of the line is read off, and is noted as a measure of the defining power of the lens in the centre of its field. The light of the lamp is regulated so that the image of the line can be seen as soon as possible.

Besides measuring the defining power where the axis of the lens cuts the focal surface, an observation is also made at a point representing the extreme corner of the plate of the size for which the lens is being examined, that is, at a distance from the centre equal to half the diagonal of the plate. As the object of this second test is to measure the general definition over the whole plate, the focus is taken at a position half-way between the point of observation and the axis of the lens, this being the method generally adopted by practical photographers when desirous of getting the best general focus. It is necessary, moreover, that the test-object should be so arranged that the steel strip makes an angle of 45° with the horizon; for, since the diffusion of the image near the margin may be due to astigmatism, a false impression of the defining power will be obtained if the image of the dark line coincides in direction with either of the focal lines; whereas if it bisects the angle between them, as will then be the case, there is no error in the result from this cause. The test is not, however, conducted in quite the same way as in the first instance; the test-object is set at a known angle, and the stops are slipped in one

after another, beginning with the largest and going on to smaller ones, until the image of the black line on the bright ground is first just visible; the C.I. No. of the stop with which the lens gives definition up to a known standard at the extreme corner of the plate is thus ascertained, and, as it may fairly be assumed that the definition will be no worse than this at any other part of the plate, it follows that the defining power over the whole plate comes up to or exceeds the standard selected.

It cannot be denied that the defining power is the most important quality of a photographic lens for almost every purpose, and yet the best method of testing definition has never been satisfactorily discussed or considered. If a thoroughly good test could be devised, it would be hardly necessary to examine at Kew for curvature of field or for astigmatism, for these defects are only hurtful in so far as they affect definition. But it must be confessed that the method above described is open to some objections, and the following discussion is merely intended to show that it is the best that could at present be devised.

In considering this question, it was natural that attention should first be turned to the excellent arrangements adopted at Kew for testing the definition of telescopes. The method generally used, especially when dealing with instruments supplied for the public service, is to compare each one separately with a standard telescope by an observation on a distant object; telescopes sent for examination can by this means be *passed* or *rejected*, but hardly classified. But in examining photographic lenses, where there is a much greater variety of form and pattern, it would be quite out of the question to keep a sufficient number of standard lenses to be of any practical use. Thus little assistance was obtained from the experiences gained in the examination of telescopes.

It was necessary therefore to seek some method which did not depend on comparisons with standards, and in devising such a test the object most to be kept in view was evidently to diminish as far as possible the errors due to the variations either in the transparency of the atmosphere or in the personal qualities of the observer.

With regard to the first point, that is, the effect of fog, mist, and dust in the air, the only way to avoid errors from these causes appeared to be to conduct this test in a room. This was considered especially necessary in a climate like that of London. It is no doubt theoretically right to examine portrait lenses, or lenses for copying plans, by observations on a test-object not too far away; but for landscape lenses a distant test-object would, from other points of view, be preferable, and the adoption of the examination in a room was only the choice of the lesser of two evils.

With regard to variations due to the personality of the observer, the

case is more difficult. Probably the most important consideration is that the test should not be based on a mere judgment, the reason for which one person cannot readily communicate to another. In many works on photography the extent of field over which the lens produces a "sharp" image is discussed, as if by mere inspection this could be determined; whereas no two people would exactly agree as to where the diffusion of the image was sufficient to be classed as want of sharpness, and no two objects would serve equally well for such a test. It is essential, at such an establishment as the Kew Observatory, that the observer should obtain some definite numerical result from his examination, even though it may be considered advisable to merely employ general expressions in the wording of the certificate; under any other system it would be impossible for any length of time to prevent the standards from varying.

Still more difficult is it to avoid errors from actual variations in eyesight, whether between different individuals or at different times in the same individual. Some general conditions may, however, be laid down. When the illumination of an object is very feeble, the subjective light of the eye, as it has been called by Helmholtz, plays an important part in determining the least intensity of illumination which is visible, and this subjective light is a very variable quantity; the eye increases in sensitiveness for a long time when light is excluded from it, the increase at first being very rapid, which may be another way of expressing the same fact. Hence, any feebly illuminated object must be a bad test-object, for its appearance will vary very materially according to the state of the eye. On the other hand, if the illumination is too bright, the eye will be much influenced by irradiation, and the subjective effect on the eye will be a bad indication of the true condition of the object; moreover, as irradiation is the effect on the appearance of an object produced by brighter surrounding objects, and as this effect diminishes as the differences of shade get less, the test-object should show no marked contrasts in illumination. But in applying these general remarks to the case under consideration, it must be remembered that it is not the test-object which is seen by the eye; it is the image of the test-object, as produced by the lens under examination. Hence, it appears that the test-object should produce an image of medium intensity of illumination, and one in which there are no great differences in shade. The test-object used at Kew, it will be remembered, consists of a perfectly black object seen against a bright background, and it might therefore appear as if it were not a good selection. In order to prove that, as a rule, the differences of shade in the image are small, and that no objections can be raised to the Kew test on theoretical grounds, it is necessary to show what is the effect on the image produced by a want of defining power in the lens.

The result of bad definition in the lens is to make the image of a point occupy a sensible area on the photographic plate, and consequently to prevent the image of the edge of a surface from being sharply indicated. The general effect can be best illustrated by means of figs. 7 and 8, where the abscissæ are enlarged dimensions measured on the plate, and the ordinates indicate the intensity of illumination at each point. In fig. 7 let *a*, *e*, *b* represent a section through the image of a small spot of light. In fig. 8 let the curve *f*, *h*, *k* represent the actual image of the edge of a bright surface, which would be represented by *f*, *d*, *c'*, *k* if the defining power of the lens were perfect; it is evident that *a' b'* in fig. 8 is equal to the limiting value of *ab* in fig. 7 as the spot of light becomes infinitely

Fig. 7.

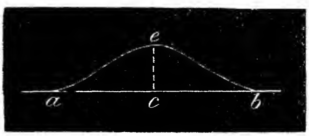


Fig. 8.

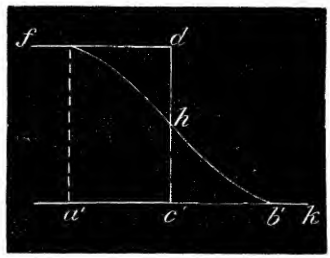
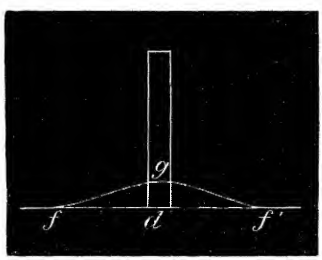
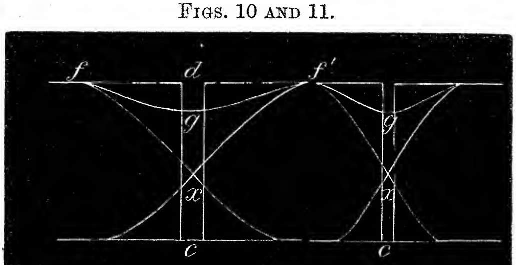


Fig. 9.



Figs. 10 and 11.



small. In fig. 10 is shown the effect of bringing two bright surfaces near together; that is to say, of a dark line as seen against a bright background; f, g, f' will represent a section through the image of the line, gx being equal to xc. If this curve is turned upside down, as in fig. 9, it can be shown that it represents the image of a bright line on a dark ground.

In this latter case—that of the bright line on the dark ground—it can be readily seen that the effect of narrowing the slit of light will be to decrease the illumination gd at the centre of the line until it becomes zero as the slit closes. The worse the definition of the lens, the sooner will the centre of the line reach the limit of visibility; but by ascertaining what is the width of the finest bright line just visible, a good test for defining power will not be obtained for the following reasons; in the first place, the illumination of the image will be feeble, which has already been shown to be objectionable, and in the second place, since with feeble illuminations the ocular sensation varies as a first approximation as the intensity of the illumination, considerable errors would arise through the difficulty of obtaining a constant illumination through lenses of different types.

These objections do not apply, however, to testing definition by finding the width of the finest dark line that can be seen against a bright background. In this case as the line becomes thinner, the illumination at its centre increases until it reaches that intensity of illumination which can no longer be distinguished by the eye from the illumination of the field. If the illumination cg in fig. 10 can be distinguished from cd by the eye, it is evident that a blurred image of the dark line is visible, and if any illumination greater than cg is indistinguishable from cd by the eye, it is evident that the figure represents the image of the thinnest black line which is visible. Fig. 11 represents generally the same condition of things as that shown in fig. 10, except that the defining power of lens is much better; and it will be seen how much finer the line must be in this case to produce the same proportional illumination at its centre; that is to say, before the limit of visibility is reached. Now there is a certain intensity of illumination at which and about which the eye is at its maximum of sensitiveness to differences of shade, and this is when the object is what would be described as not bright and not dark; between these wide limits the minimum difference of shade visible is a fixed proportional part of the total illumination. This proportion differs with different observers, but not to a very great extent. Hence if a plan is adopted by which a dark line on a bright ground can be made to vary in thickness, and if the illumination is arranged so that the eye is at its maximum sensitiveness (that is therefore so that the line remains longest visible as it diminishes in width), then the moment at which it disappears will occur when the

difference of intensity of illumination of the centre of the line and the field is the minimum difference of shade discernible by the eye, and this will be independent of the actual intensity of the field, and will not vary much with different observers. But it has been shown that the thickness of the line does vary with the defining power of the lens, and it may therefore be concluded that the test adopted at Kew is not open to serious objections on theoretical grounds.

In the foregoing discussion, it has, however, been assumed that the curve representing the image of the edge of a surface is such as that which Helmholtz has shown to be produced as an ocular effect by the circles of diffusion being due to want of accommodation of the eye itself:* it will be observed that no part of the curve is tangential to the vertical. If, however, the curve is similar to that given by the

FIG. 12.

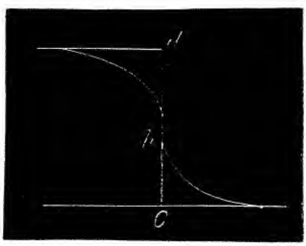
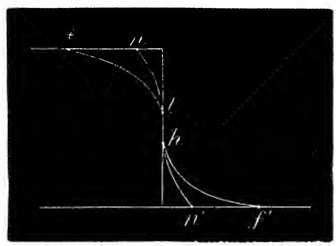


FIG. 13.



same author as being due to dispersion in the eye, and illustrated in fig. 12, it will be seen that the result of gradually diminishing the thickness of a line will not be exactly as above described; for, however thin the dark line on the bright ground becomes, the intensity of illumination at its centre can never be more than twice *ch*; and if the ratio of twice *ch* to *cd* is less than a given ratio, the image of the black line will remain visible until it is so thin that the eye cannot perceive it. Therefore it might come about that two lenses giving images of the edges of surfaces as different as *flhf'* and *nlhn'*, as shown in fig. 13, might give equally good results under the Kew test for definition, because in both cases the limit of visibility would be due to the minimum size of the line visible by the eye, and would have nothing to do with the definition of the lens. Helmholtz remarks on the very little evil effect of a diffusion represented by the curve shown in fig. 12, since the true edge is always visible. Hence we may assume that the Kew method still gives in such cases a good practical test for definition, though it does not test the amount of dispersed light over the image of fine lines, or, as a photographer

* 'Optique Physiologique,' Helmholtz, Paris, 1867, p. 185.

would say, the brilliancy of small objects. In fact, since the definition of an objective could only be rigorously expressed by a curve (or, more accurately, a surface) with dimensions, it is impossible for any one result to give all the information on this head which might be desirable.

As the eye is capable of detecting a difference of shade of about 1 per cent. of a moderately illuminated field, it will be only necessary for the curve shown in fig. 12 to be tangential to the vertical for 1 per cent. of its height to render the image of an infinitely thin line visible in so far as that visibility depends on difference of shade. But take the case of a line not absolutely black, and seen against a bright background; then, in fig. 10, the illumination of the centre of the image will be represented by *gc*, *plus* some proportional part of *gd*; in comparison with the case of the absolutely black line, it can be shown that the curve must be tangential to the vertical for a proportionately greater distance before the shade of the centre of the image of the infinitely thin darker line will be sufficiently deep to form a visible contrast. For instance, if the line is illuminated to nine-tenths of the intensity of illumination of the field, the curve must be tangential to one-tenth of *dc* (see fig. 10) before this condition of things occurs. A test depending on the thickness of a line which is darkened to a definite proportional intensity of the field would therefore present this disadvantage, that there would be fewer occasions on which different degrees of imperfection of definition of lenses would show the same result in testing; such a test may therefore in future be adopted at Kew.

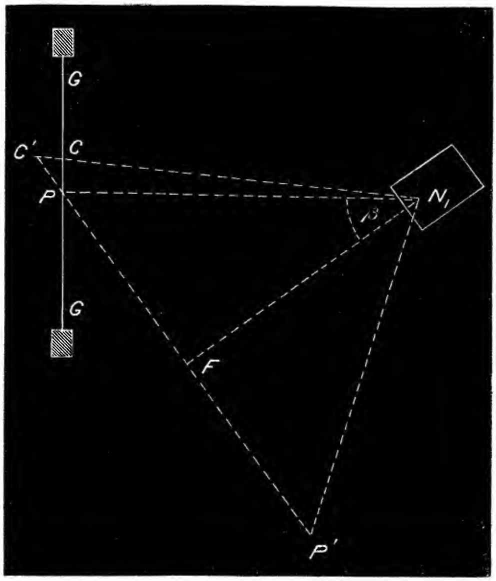
It should, however, be remarked that in the whole of the above reasoning it has been assumed that the minimum proportional difference of shade visible is the same in a thin line as in a thick one, which can hardly be the case. But this false assumption will not, it is thought, vitiate the general conclusions arrived at.

It is of course conceivable that the actinic rays will be brought to either a better or to a worse focus than the visible rays; it is believed, however, that no serious error is likely to result from the test being done by the eye and not by photographic methods; it is almost certain that the curve representing the edge of a surface will have the same general character in the two cases, and therefore that the results obtained with the eye will be a good indication of those which would be obtained by photography.

14. *Distortion. Deflection or sag in the image of a straight line which, if there were no distortion, would run from corner to corner along the longest side of a —— by —— plate = 0·—— inch.*

The following is the method adopted at Kew of measuring the distortion produced in the image by the lens under examination. Let fig. 14 be a vertical section through the testing camera; GG re-

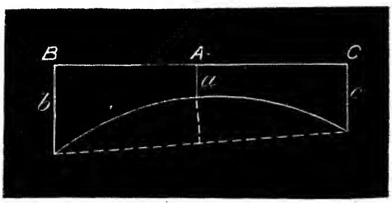
FIG. 14.



presenting the ground glass; F the principal focus; and N_1 the horizontal axis, which passes through the nodal point of emergence, the adjustment for that purpose having already been made for test No. 10. The lens-holder carrying the lens is first turned in either direction through an angle β, such that PF, or $FN_1 \tan \beta$, or $f \tan \beta$ is equal to half the *shortest* side of the plate for which the lens is being tested. (The *horizontal* movement of the swinging beam in the testing camera gives an easy means of determining the angle β; a distant object is first brought to focus at the centre of the ground glass, and then the swinging beam is revolved about the axis A (see fig. No. 1) until the image has moved along the graduated scale a distance equal to half the shortest side of the plate; the beam is thus made to move through the angle β, which can be read off with sufficient accuracy on BC, the top of the wooden stool, which is graduated for that purpose). After this adjustment has been made,

the ground glass is brought into focus by observing the image of a distant object at a point P, a little below C, the line engraved on the glass; under these circumstances, if the principal focal surface is a plane, and if the lens were being used in the ordinary manner, PP′ would be the position occupied by the photographic plate, the section shown being taken across the centre of the plate parallel to its shortest side. The small distance PC is carefully measured; this length is then multiplied by secant β, thus obtaining C′P, which we will call a. The swinging beam is now revolved about the pivot in either direction, so that the image moves along the scale on the ground glass a distance equal to half the *longest* side of the plate for which the lens is being examined; the sketch in fig. 7 is still more or less applicable, PP′ still representing a section across where the photographic plate ought to be, but this time at the end of the plate, not at its centre; (F, therefore, no longer represents the principal focus); in fact, what has been done is to make the image describe what, neglecting distortion, would be a straight line from the centre to the corner along the longest edge of the plate: after this movement has been made, the length of C′P is again obtained by measurement and calculation, and this time let the result be called b; the operation is repeated when the swinging beam is revolved to an equal angle on the other side of zero, and a third length, c, is thus obtained. In fig. 15, let BAC be equal in length to the longest side of the plate,

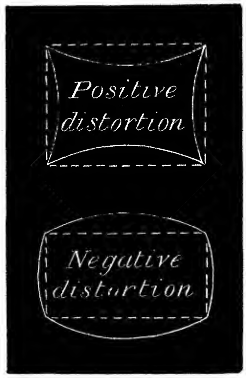
FIG. 15.



and let a, b, and c be the lengths just obtained; then the curve bac will evidently represent the image of a straight line thrown by the lens under examination along the edge of the longest side of the plate. Since the image travels along a line very nearly parallel to the engraved line on the ground glass, BAC will be nearly parallel to the chord of the curve, and $\frac{b+c}{2}-a$, which is the length recorded in the Kew certificate, will be a very close approximation to the sagitta or sag of the curve.

The image of a rectangle near the limits of a photographic plate will appear, when any distortion is visible, like one or other of the forms indicated in fig. 16. The sagitta is conventionally considered

Fig. 16.



positive if it is measured towards the centre of the plate from the chord, thus giving the name of positive and negative distortion in the two cases.

The distortion for distant objects is not necessarily exactly the same as for nearer ones, and therefore the uses for which the lens is intended should not be forgotten; for example, with portrait lenses, an object some 10 to 20 ft. away should be used to throw the image in the above test.

Probably it will not at once be admitted that this is the best means of measuring distortion; for no doubt it might be done in many other ways, and a method might easily have been selected which would have been less open to criticism on purely scientific grounds. We believe, however, the Kew certificate gives the information really required in practice. In order to determine if a lens is suitable for any particular purpose, all that is required to be known is whether the image of a straight line near the edge of the plate will show too much curvature, the amount of tolerance depending greatly on the work for which the lens is to be used. There is no means of enabling the photographer to form a judgment on this point more readily than by giving him the sagitta or sag in the image of a straight line along the edge of his plate. That it would be difficult to find a better method may, perhaps, be made more evident with the aid of figs. 17 and 18, the former representing a section through a lens and the photographic plate, and the latter showing part of the plate in plan, with the curved image of a straight line just inside its margin. In fig. 17, let N_1 be the nodal point of emergence; S_β the centre of similitude for rays emanating from a distant object and making an angle β with the axis; and S_θ the same for an object at an angle θ; e and g will, therefore, be the images of these two objects as seen on the plate, whereas, if there had been no distortion, they would have appeared

Fig. 17.

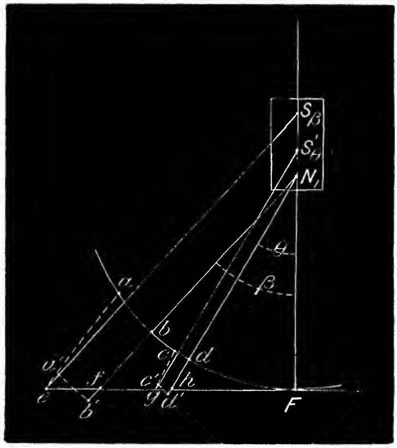
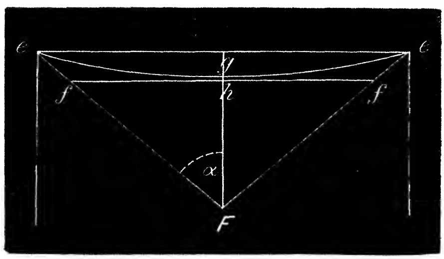


Fig. 18.



at f and h respectively; ef and gh will, therefore, be the total distortion in each case. In fig. 18, let the rays coming from the objects, of which the images are seen at e and g, make the angles β and θ with the axis of the lens at the nodal point; if ef and eg are equal in length to the lines similarly denoted in fig. 17, it is evident that the curve ege represents the image of a straight line, which, if there had been no distortion, would have appeared as the line fhf. Now it would not have been difficult to have devised means of measuring the total distortion at any part of the plate; for instance, to have measured the distortion ef for the point e at the corner of the plate—but the following considerations show, it is thought, that that would not be a suitable way of testing the lens; let the curve efe in fig. 18 represent the greatest curvature that would be tolerated for the class of work for which the lens is intended; compare the lens producing this curve with another in which S_β occupies the same position, but in which S_θ is nearer the

nodal point N_1; *ef* would be the same in the two cases, but *gh* would be less in the second case, and the curvature would therefore exceed the tolerated limit: with two lenses giving an equal total distortion at the margin, one should be passed and the other rejected. The total distortion at any one point will not therefore give a measure by which the lens should be judged, the greatest rate of change in the distortion more nearly representing what is required to be known; and, as the rate of change is certain to be greatest at the margin, the Kew certificate supplies the information required.

The tourniquet has already been mentioned as an apparatus which has been specially recommended for the purpose of testing photographic lenses; by means of this invention, Commandant Moëssard obtains an excellent means of detecting distortion, but hardly of measuring it in a way to indicate the curvature produced in an image. It will be remembered that the lens can be revolved about an axis which passes through the nodal point N_1, whilst the eyepiece remains stationary; the effect of this movement can be seen in fig. 17 by imagining the lens to be stationary, whilst the object and the eyepiece revolve about the nodal point, the arc *abcd*F being the path traversed by the eyepiece. Let *a* be the image of the object after the lens has been revolved through an angle β, and *e* the position where the image would be seen on the photographic plate; for there is no reason to believe that the line *ea* will coincide exactly with the line eS_β; if there were no distortion, *b* would be the image as seen in the tourniquet, and the distance moved by the image from *b* to *a* is what is measured by that apparatus. It will be noted that the image *a* will be much out of focus if the lens has a fairly flat field; and that, after re-focussing, *a′* will represent the image, and *b′* the point from which the measurement is taken; this re-focussing will tend to reduce any error which may be due to *ae* not being coincident with eS_β, but such a movement in the middle of an operation is rather objectionable on mechanical grounds. Putting this objection aside, it will be seen that we do not get a ready means of finding the curvature produced in the image as seen in plan in fig. 18; for, if *c′d′* is the length measured by the tourniquet when the lens is revolved through an angle θ, then the sagitta of the curve is equal to

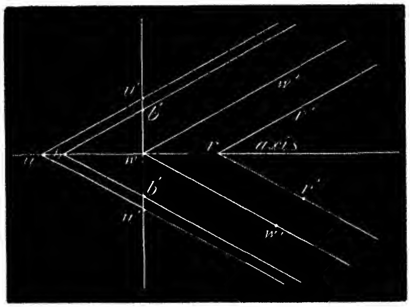
$$a'b' \sec \beta \cos \alpha - c'd' \sec \theta.$$

15. *Achromatism. After Focussing in the Centre of the Field in White Light, the Movement necessary to bring the Plate into Focus in Blue Light (dominant wave-length* 4420*),* = 0· —— *inch. Ditto in Red Light (dominant wave-length* 6250*),* = 0· —— *inch.*

The photographer may be said always to adjust his focus in daylight, and if the actinic rays are not brought to the same focus as the

dominant rays for white light, the definition obtained in the photograph itself cannot be perfect. In fig. 19, let *u'wu'* be the position of

FIG. 19.



the photographic plate, the focus of which has just been adjusted in daylight; if the lens has not been properly corrected for achromatism, the different coloured rays will form different cones, and those coming to a focus at *w* will have a wave-length of about 5570, for that, I am informed by Captain Abney, is generally speaking the dominant wave-length for white light. Let *b'bb'* be the cone of rays of 4420 wave-length, which is not far from the position of the maximum actinic effect for ordinary bromide dry plates, and let *u'uu'* be the cone for rays of 4000 wave-length; since the actinic effect with silver salts begins to fall off rapidly at about 4000 wave-length, the cones outside the cone *u'uu'* may be neglected, and it may be taken that the image of a point covers a disc on the photographic plate of which *u'w* is the radius. It is evident that what the photographer wants to know, with regard to the achromatism of his lens, is the amount of diffusion caused in the image by any errors in its construction, that is to say, what is the actual size of the disc of diameter *u'w*.

The examination for achromatism is therefore made in the following manner:—First the focus is carefully adjusted in daylight on a suitable object placed as far away as possible in the room, and then the focus scale is read off. After this, a sheet of blue glass, the colour of which has a dominant wave-length of 4420, is placed behind the object and close in front of a small opening in the shutter through which all the light enters the room; the focus is re-adjusted, the focus scale read off again, and the difference in reading to that observed in white light is noted; the length *bw* in fig. 19 is thus obtained. Now let f be the principal focal length of the lens; and let f' be the focal distance when the observation was made, which can be easily obtained with sufficient accuracy by a direct measurement

from the ground glass to the nodal point of emergence, or to the pivot which has been made to pass through that point. The difference of focus, bw, noted between the blue and white light is then multiplied by f/f', and the result thus obtained is that finally recorded in the Certificate of Examination as if it were the direct result of an observation made on a distant object. Exactly the same process is then repeated with a sheet of red glass, the colour of which has a dominant wave-length of 6250.

The reason for multiplying the result of the observations by f/f' is that it would evidently be unfair to test objectives of different focal lengths on a near fixed object, for in some cases the ground glass would be close to the principal focus, and in others far removed from it. It seems, therefore, advisable to reduce all results so as to make them equivalent to observations taken on infinitely distant objects, and this is done by applying this correction. An assumption is here made that the difference of focus between different coloured rays in the same lens varies directly as the focal distance, and this in all probability, though not strictly accurate, introduces an exceedingly small error in the results.

The blue and red glasses, which were selected and measured for colour by Captain Abney, form a perfect contrast, as may readily be seen by placing them together and observing how very nearly completely all light is excluded.

By simply noting the difference of focus recorded in the certificate between observations made in red and white light, or between observations made in blue and white light (the latter being of far more practical importance), it can at once be told if the lens is or is not well corrected for achromatism. But it would seem desirable, as already remarked, to form an estimate of the actual amount of diffusion produced in the image as a result of any error that may be detected in the chromatic adjustment of the lens. Now there can be no difficulty in determining the size of the disc of radius $b'w$, for bw has been directly determined by experiment, and, since the cone $b'bb'$ represents the cone of rays of maximum actinic effect, on this disk will be concentrated the bulk of the rays which produce the effect on the photographic plate. But what we want in reality to find is the radius $u'w$, since that has been shown to represent more accurately the radius of the disc of diffusion; it may, however, be remarked that no fault can be found on this head with the method of testing, because the probabilities of error are lessened by taking the observation with rays of the maximum actinic effect. With a lens not at all corrected for achromatism the length between the different foci for different coloured rays varies approximately as the difference of the squares of the wave-lengths of the colours in question; and, taking the wave-lengths as above given, uw will be found to be to bw as 5 to

4. But it must be confessed that this rule may have little or no relation to the truth with a corrected lens, and it is merely adopted as the only approximation obtainable. It is assumed, therefore, that $uw = 5/4\,bw$. Let bw, the result obtained by the examination for achromatism, $= \alpha$; let the diameter of the disc of confusion, or twice $u'w$, $= \delta'$; let the principal focal length of the lens $= f$; and let the effective aperture $= \epsilon$. Then it can be seen, by reference to fig. 6, that—

$$\alpha = \tfrac{4}{5}\frac{f\delta'}{\epsilon} = 8\,\delta'\sqrt{(\text{C.I. No. of stop})}.$$

The table on p. 427, which gives the values of $20\,\delta\sqrt{(\text{C.I. No. of stop})}$, affords a ready means of obtaining the required results in the following manner:—

Knowing the C.I. No. of the stop, decide on δ', the diameter of the maximum disc of diffusion that will be tolerated; then, under the columns thus ascertained, look out μ in the table, multiply the figure there given by $\frac{2}{5}$, and the maximum difference of focus, α, that can be tolerated between white and blue rays is thus obtained.

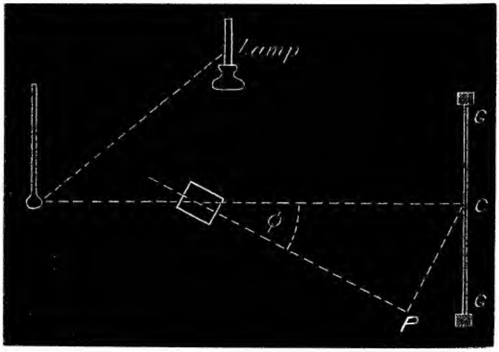
Or, in the line opposite the stop of the size under consideration, find a number equal to α, the observed difference of focus for white and blue rays; then δ', the diameter of the disc of diffusion, will be 5/2 times δ, the figure given at the top of the column in which α has been found.

It may be observed that the either the principal focal length or the position of the nodal point of emergence may vary as different coloured lights pass through a lens. It would not be difficult to investigate these two sources of error separately, but the results would be of little or no practical value.

16. *Astigmatism. Approximate Diameter of the Disc of Diffusion in the Image of a Point, with stop C.I. No.—— at —— inches from the centre of plate = 0· —— inch.*

The following is the method of examination for astigmatism:—The room is darkened, and in front of the lens is placed a thermometer bulb, thus obtaining, by means of the reflection of the light of a small lamp, a fine point of light. The lens holder of the testing camera is revolved upwards or downwards about the horizontal axis so that the axis of the lens makes an angle, ϕ, with the path of the rays coming from the thermometer bulb; the angle ϕ is such that the point of observation represents the extreme corner of the plate of the size of which the lens is being examined; that is to say, if, in fig. 20, GG represents the position of the ground glass, then CP is equal to half the diagonal of the plate; this angle has already been found for previous tests. If the lens shows any astigmatism, the image of

FIG. 20.



the point of light can be made to appear, first as a fine vertical line, and then, as the focus is lengthened, as a fine horizontal line. The focal scale is read off at each of these positions, and the difference, γ, between the two readings gives a measure of the astigmatism. But, in order to judge of the amount of astigmatism that can be tolerated, the diameter, σ, of the disc of diffusion caused thereby should be calculated. This is done by multiplying γ, the difference of focal distance of the focal lines, either by $\frac{f}{f'^2}\frac{\epsilon\cos\phi}{2}$ or by $\left(\frac{f}{f'}\right)^2\frac{\cos\phi}{20\sqrt{(\text{C.I. No. of stop})}}$, where ϵ is the effective aperture, f the principal focal length of the lens, and f' the focal distance when the observation was made. As the thermometer bulb is placed at the same distance from the testing camera as was the object in the examination for achromatism, the ratio f/f' is exactly the same as in that case. The same result may be obtained by the use of the table on p. 427 in the following manner:—Find the value of δ, the diameter of the disc of diffusion, on the supposition that the μ of the tables has the value just obtained for γ; multiply the value thus obtained for δ by $\left(\frac{f}{f'}\right)^2\cos\phi$, and we get σ, the required value of the disc of diffusion due to astigmatism. This is the quantity recorded in the Certificate of Examination.

That this is the case can readily be seen by reference to fig. 21. Here AB represents the effective aperture, F_1 and F_2 the positions of the focal lines, and PH the position that the photographic plate would occupy. At F_2 the image appears as a fine line perpendicular to the plane of the paper, and at F_1 it is represented by the line ab; half way between these two points the rays cut the plate in the form of a disc, of which $a'b'$ is a diameter. Any movement of the plate from this position must lengthen out the disc of diffusion in one

Fig. 21.

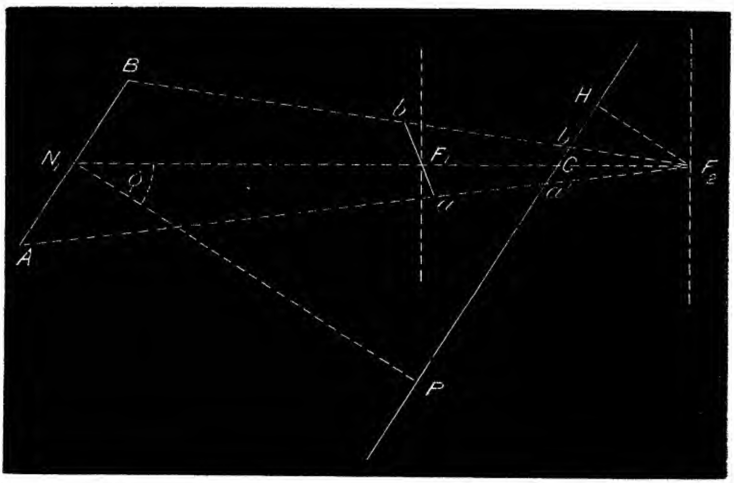
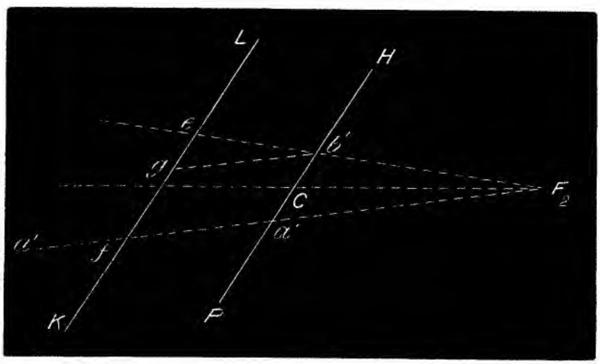


Fig. 22.



direction or the other, and this, therefore, is the position that the photographer naturally adopts as his focus. By similar triangles—

$$a'b'/\mathrm{AB} = \mathrm{F_2C}/\mathrm{F_2N_1} \text{ and } \mathrm{F_2C}/\mathrm{CN_1} = \mathrm{F_2H}/\mathrm{N_1P}.$$

Therefore, since $\mathrm{CN_1}$ and $\mathrm{F_2N_1}$ are approximately equal,

$$a'b' = \mathrm{AB} \,.\, \mathrm{F_2H}/\mathrm{N_1P}, = \frac{\epsilon}{2f'} \cos\phi \; \mathrm{F_1F_2}.$$

Now $\mathrm{F_1F_2}$ represents γ, the movement of the ground glass, which was the measurement recorded. In the case of the examination for achromatism, it was shown that it was unfair to conduct the test on a near

object without applying a correction, so as to make the result equivalent to an observation on a distant object, and that this correction could be made by multiplying the measurement recorded by f/f'. For the same reason, $a'b'$ must be multiplied by f/f' in this instance to obtain the true value of σ. Thus—

$$\sigma = \frac{f}{f'^2}\frac{\epsilon\cos\phi}{2}\gamma = \left(\frac{f^2}{f'}\right)\frac{\cos\phi}{20\sqrt{(\text{C. I. No. of stop})}}\gamma.$$

In considering the combined effect of astigmatism and curvature of the field, it should be remembered that it has been assumed that the photographer would focus his plate in the position PH, as shown in fig. 21, and that the principal focal surface, PH, was a plane; this is, however, never the case. If the focal surface is curved, it is evident that the best general focus is obtained by observing the image of an object at a position about half way between P and C on fig. 21. In fig. 22, which is part of fig. 21 enlarged, let KL be the position of the plate when focussed in this manner, the distance between KL and PH being, therefore, due to the curvature of the field. Through b' draw $b'g$ parallel to F_2a'; then, by comparing this figure with fig. 6, it will be seen that eg in both cases represents the diameter of the disc of diffusion due to the curvature of the field. Since, in fig. 22, fe represents the longest diameter of the ellipse of diffusion due to the combined effects of curvature and astigmatism, and since it is equal to the sum of $a'b'$ and ge, the diameters of discs of diffusion due to these two causes taken separately, it will not be unfair to look upon the evil effect of astigmatism as a simple addition to the evil effects of curvature. In using the table given on p. 427 in the manner described on p. 443, it would therefore be better if we subtracted the diameter of the disc of diffusion due to astigmatism from the diameter of the maximum disc of diffusion which is to be tolerated, and used the difference as the δ in the table; we should thus get a more correct notion of the size of the stop that could be used to obtain any required standard of definition. The objection to this use of the table is that the astigmatism, that is, the distance F_1F_2, varies to a certain extent with the size of the stop used during the observation.

In the above discussion it has been assumed that the focal lines are sharply defined. If this is not the case the reasoning here given is defective, because the distance separating the focal lines is then no indication of the amount of diffusion. An exaggerated idea of the amount of diffusion due to astigmatism may thus be obtained by the above method of calculation, for the disc may have only half the diameter thus found. Therefore, unless the focal lines are sharp—that is, unless the image of a point appears as a *very* thin line, first in one direction and then in another—no entry is made in the certificate.

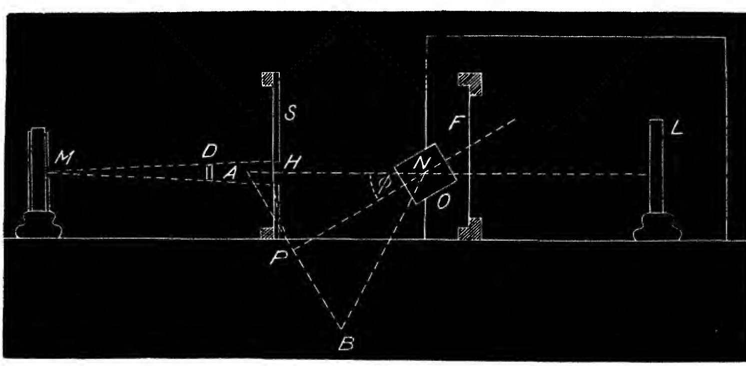
Objections have been raised to the use here made of the term *astigmatism*, when it is intended to mean the effect of spherical aberration on oblique rays; it has been proposed to limit the use of the word so as merely to signify cylindricity in the lenses, such as might be produced by turning them in a lathe with elliptical motion. Whatever may be the theoretical value of this objection, we fear that the use of the term has been so thoroughly incorporated into the photographic vocabulary, both in England and abroad, that it would now be impossible to substitute another expression in its place.

17. *Illumination of the Field. The figures indicate the relative intensity at different parts of the plate.*

With C.I. Stop No. .			With C.I. Stop No. .	
At the centre.............	100	:	Ditto..................	100
At inches from the centre		:	Ditto..................	
At inches from the centre		:	Ditto..................	

The intensity of illumination of the field is always greatest near the axis of the lens, and falls off more or less rapidly towards the edges of the plate. The lens should therefore be examined with the view of ascertaining if this inequality of illumination is greater than that which experience shows must be tolerated under given circumstances. The apparatus employed for conducting this test is shown in fig. 23, the method being devised by Captain Abney. There is a

Fig. 23.



fixed lamp, L, the position of which is not changed during the observations; F represents a paper screen, placed in that position in order to give a practically uniform source of light; O is the lens, which is fixed in a frame, not shown in the sketch, revolving about the pivot N; by means of a suitable adjustment, this axis, N, is made to pass through the nodal point of emergence of the lens. At S there

is a sheet of cardboard with a small hole in the centre at H, and this screen, hole and all, is covered with thin white paper on the side away from the lens; the distance between H and N is always made equal to the principal focal length of the lens; the bar D is made to cast a shadow from the movable lamp M on the paper just over the hole in the cardboard; thus, in this shadow, the paper is illuminated entirely by transmitted light from the lens, whilst the paper round it is illuminated entirely by the light of the movable lamp.

An observation is made in the following manner:—The lens is first placed in such a position that its axis passes through the hole H; the lamp M is then moved backwards or forwards until the transmitted illumination of the paper at H is made to match as nearly as possible the reflected illumination of the paper round it; the distance between S and M is then noted. The lens is now placed in the position shown in fig. 23, where AB represents the length of the diagonal of the plate for which the lens is being examined, and where the angle ϕ is half the angle of field under examination. The balance of light is readjusted by a movement of the lamp, and the distance MS is read off a second time. By finding the inverse ratio of the square of these two readings, we thus obtain the ratio between the illuminations at P and H, the lens being in the position shown in the sketch, and the object being supposed to be equally illuminated in both cases. But what is wanted is the ratio between the illuminations on the plate at P and A; this is found with perfect accuracy by multiplying the ratio of the illumination at P and H, as above obtained from the observations, by $\cos^3 \phi$, and this result is that which is entered in the Certificate of Examination. The relative illumination of the centre and of any part of the field can, of course, be obtained in this manner, in the above instance the corner of the plate being the point chosen.

This test may with advantage be made with the largest stop supplied, and also with the stop which has been shown, under test No. 13, to give good definition over the whole plate.

It cannot, however, be denied that there are objections to this method of examination. The fact that the illumination of the plate is not uniform is due to several causes:—(1.) The amount of light which passes through any aperture evidently diminishes with the obliquity. (2.) With lenses not free from distortion, the effective aperture itself varies with the angle of incidence. (3.) The amount of reflection from the surfaces of the lenses, and consequently the amount of transmitted light, varies with the angle of incidence. The method of observation above described may be said to fully take into account these three causes of variability in intensity. Then again (4) the light falling on the plate varies inversely as the square of its

distance from the nodal point, and also (5) with the obliquity with which the rays strike the plate. As far as these two latter considerations alone are concerned, it is evident, therefore, that the illumination on the plate varies as the third power of the angle of incidence, and also that by multiplying the result obtained on the screen at H by $\cos^3 \phi$ we obtain the required result on the plate at A. Thus the record in the certificate includes all these first five causes of irregularity of illumination. But there are other causes which are not correctly represented in this method of examination. In lenses not free from distortion the nodal point of emergence varies in position with the angle of incidence, and as the pivot N does not shift its position with reference to the objective during the observation, the condition of illumination of the photographic plate cannot be accurately represented. This is probably a trifling cause of inaccuracy; but one somewhat serious source of error remains to be mentioned. The method of examination does allow for (6) the variation of illumination due to the different amount of glass through which the oblique pencils have to travel; but, as the observation is made by eye, no allowance can be made for the fact (7) that the actinic rays may be affected in this manner out of all proportion to the apparent variations produced in the visible rays.

The method of examination adopted at Kew assumes that the light transmitted through the paper, as well as that reflected from the paper, varies in amount with the intensity of the incident light. Captain Abney informs me that his experiments prove this to be the case. But in making the observation the eye should be placed in the same position during both readings; for we have no reason to suppose the transmitted and reflected lights vary in the same way with the angle of vision.

It is impossible to suppose that the screen F will be illuminated with perfect regularity, even near its centre, and this must be a source of error, though probably a negligible one. When the axis of the lens passes through H the rays which are brought to a focus at that point, will be parallel to each other as they enter the lens; but when the axis of the lens is inclined this cannot be the case, for H will no longer be on the principal focal surface; the screen F should therefore be brought as near the lens as possible, as by that means the part of the screen from which the light comes will be more nearly identical in the two cases. The lamp L should, moreover, be placed as far from the screen as practicable, so as to make the illumination as uniform as possible. With lenses in which the nodal points are some distance apart, the part of the screen from which the light comes will vary considerably with the inclination of the axis, and considerable errors might be introduced by uneven illumination of the screen.

In deciding on the quality of a lens as regards the illumination of the field, this test should be considered in connexion with test No. 10, under which heading are given the angles of the cones of illumination. With regard to the normal use of any lens, except perhaps such as are specially designed for portraiture, certainly the whole of the smallest stop, and, as a rule, the whole of the largest normal stop, should be visible from the whole of the plate; for if the plate extends much beyond the limits of the inner cone (outside which the aperture begins to be eclipsed) the falling off of density near the edges of the plate will be a serious defect in the photograph. When considering the part of the field lying within this inner cone, it is to be noted that, the wider the angle which the lens covers, the greater is the inconvenience caused by the diminished density near the margin; if the stop is in front of or behind all the lenses, the intensity of illumination of different parts of the plate can be shown in this case to vary approximately as the fourth power of the cosine of the angular distance from the axis of the lens, and in cases where the stop is between two lenses, the limits of variation will be the third and fourth powers of the cosine of the angle. The following table is therefore inserted to give an approximate idea of the decrease of illumination as we recede from the axis of the objective, the truth lying *theoretically* somewhere between the two limits here given:—

ϕ.	$\text{Cos}^3\,\phi$.	$\text{Cos}^4\,\phi$.
0°	1·00	1·00
5	0·99	0·98
10	0·96	0·94
15	0·90	0·87
20	0·83	0·78
25	0·74	0·67
30	0·65	0·56
35	0·55	0·45
40	0·45	0·35
45	0·35	0·25

Eminent lens makers have spoken of the illumination produced by their lenses as being uniform from the centre to the margin, but our experience is that the decrease is even more rapid than here indicated. The above table shows how very objectionable is the use of wide angle lenses, whenever they can possibly be avoided. It shows, moreover, that the theoretical exposure for different stops should be materially modified according to the angle which the lens covers; for instance, taking the last column to represent the truth, it would be right, even though the stops in the two cases had the same C.I. number, to give half as much exposure again with a 90° objective as

with one only covering 40°, in order to get the same mean exposure over the whole plate.

In connexion with this test it may be mentioned that the most serious omission in the Kew examination is, that there is nothing to show the actinic transparency of the glass. A slight yellow tinge in the lenses, which would not be noticed by the eye, might yet be sufficient to seriously affect the rapidity of the objective. But no test could be devised to investigate this point which did not introduce photographic methods, and, as already stated, the consideration of expense put such operations out of consideration for the present. I should like, if possible, to have introduced some test which would have at the same time indicated the actual rapidity of the lens, and also the actual falling off of density towards the margin of the photograph; with the aid of photography this would not have been difficult, and a plan of this kind would have been adopted, but for the cost. This subject is, however, still under consideration by Captain Abney.

January 19, 1893.

The LORD KELVIN, D.C.L., LL.D., President, in the Chair.

A List of the Presents received was laid on the table, and thanks ordered for them.

The Bakerian Lecture was delivered as follows:—

Bakerian Lecture.—"The Rate of Explosion in Gases." By Harold B. Dixon, M.A., F.R.S., Professor of Chemistry in the Owens College, Manchester. Received July 8, 1892.

(Abstract.)

1. Berthelot's measurements of the rates of explosion of a number of gaseous mixtures have been confirmed. The rate of the explosion wave for each mixture is constant. It is independent of the diameter of the tube above a certain limit.

2. The rate is not absolutely independent of the initial temperature and pressure of the gases. With rise of temperature the rate falls; with rise of pressure the rate increases; but above a certain *crucial pressure* variations in pressure appear to have no effect.

3. In the explosion of carbonic oxide and oxygen in a long tube,

Figs. 1 and 2.

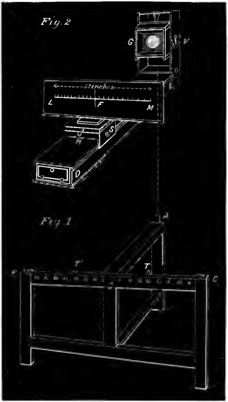


FIG. 3.

Fig. 4.

FIG. 5.

FIG. 6.

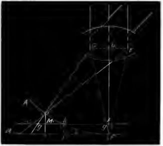


FIG. 7.

FIG. 8.

FIG. 9.

FIGS. 10 AND 11.

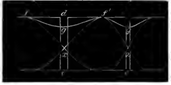


FIG. 12.

FIG. 13.

FIG. 14.

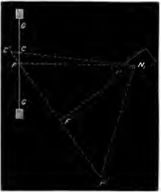


FIG. 15.

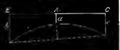


Fig. 16.

FIG. 17.

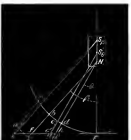


Fig. 18.

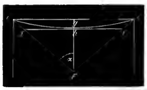


Fig. 19.

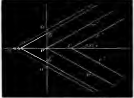


FIG. 20.

FIG. 21.

FIG. 22.

Fig. 23.